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Commitment through risk

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ABSTRACT

We show that risk-averse hyperbolic-discounting agents can benefit from positive exposure to risk and thus behave as if risk-loving. When the benefits of costly effort are delayed, selecting some risk concerning the outcome of one's own effort can serve as an intrapersonal commitment device for exerting higher effort, thereby attenuating the negative effect of time-inconsistency. Comparing the effects of time-inconsistency, risk aversion and prudence, we formulate an intuitive condition for risk exposure to be an optimal strategy and discuss several applications of this result.

1. Introduction

A risk-averse agent should not prefer a risky project over a risk-free project if ex-ante expected returns are identical. However, a typical example for the phenomenon of (excessive) risk taking, which contributes to the massive failure rates of new business ventures, is found in the behavior of business entrepreneurs. Explanations for undertaking highly risky projects range in the economics literature from costly learning under imperfect information (Brocas and Carrillo, 2009) to behavioral explanations such as biased beliefs or overconfidence of entrepreneurs (e.g. Camerer and Lavallo, 1999).

In this note, we offer a novel explanation of why economic agents may deliberately seek to increase the uncertainty about the results of their effort. We show that selecting a risky project (or increasing the amount of risk exposure associated with a project) may be used as an intrapersonal commitment device by risk-averse hyperbolic-discounting agents for exerting higher effort. It is well known that an agent's self-control problem leads to an inefficiently low effort provision as long as effort costs precede its benefits. But committing to a high-risk project can be a fully rational decision for time-inconsistent agents: if risk aversion decreases with wealth, a higher risk increases the effort, thereby attenuating the negative effect of time-inconsistency.

The modeling approach is related to two different strands of research. First, there is an extensive literature on time-inconsistency and self-control problems (see, e.g. Strotz, 1956; Thaler and Shefrin, 1981; and, more recently, Gul and Pesendorfer, 2001). Commitment devices already analyzed in the literature include investments in illiquid assets (Laibson, 1997), self-imposed deadlines (Akerlof, 1991), strategic ignorance (Carrillo and Mariotti, 2000), informational peer effects (Bataglini et al., 2005), and binding financial agreements (Ambec and Treich, 2007). We add risk-taking behavior to this list of commitment devices. The paper most closely related to ours is Salanié and Treich (2006), in which prudence is used to demonstrate that time-inconsistent agents may save earlier instead of procrastinating; their commitment result is based on the complementarity of saving decisions in different periods rather than on risk taking itself. Second, our analysis applies recent concepts from the risk and uncertainty literature, in particular the impact of prudence (Kimball, 1990; Eeckhoudt and Schlesinger, 2006) on optimal prevention (Eeckhoudt and Gollier, 2005; Menegatti, 2009; Dionne and Li, 2011). In our context however, prudence can lead to deliberate risk-seeking behavior, i.e. the opposite of prevention.

2. The model and results

Consider a time-inconsistent agent who makes two decisions in two consecutive periods and receives the benefit in the third period

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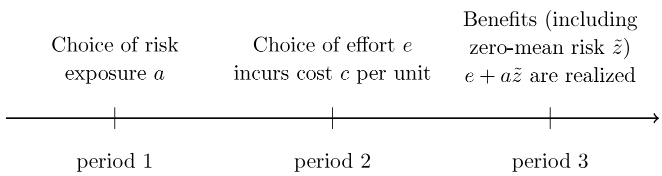

Fig. 1. Timing of the decision problem.

(see Fig. 1). In period 1, the agent decides about the amount of her exposure to some risk $a\tilde{z}$ by choosing $a \geq 0$, where $\tilde{z}$ is a zero-mean random variable such that $E(\tilde{z}) = 0$, $E(\tilde{z}^2) = 1$. In period 2, she selects a level of effort $e \geq 0$, incurring constant marginal cost of effort c. In period 3, she receives the benefit $e + a\tilde{z}$, which depends on her effort and the realization of the additive risk. Note that by selecting a higher (lower) level of risk exposure a in the first period, the agent increases (decreases) the volatility of the benefit without changing its expected value. The choice of a can also be interpreted as an investment in the riskiness of a project; that is selecting $a = 0$ corresponds to choosing a risk-free environment.

Following Laibson (1997), the intertemporal preferences of the agent are in period 1,

$$-ce + Eu(e + a\tilde{z}), \tag{1}$$

and in period 2,

$$-ce + \beta Eu(e + a\tilde{z}), \tag{2}$$

where $u(\cdot)$ denotes the period 3's instantaneous utility function. Furthermore, we assume that $u'(\cdot) > 0$ and $u''(\cdot) < 0$, and hence the agent's degree of absolute risk aversion is positive, $AR \equiv -u''(\cdot)/u'(\cdot) > 0$. The hyperbolic-discounting parameter β, with $0 \leq \beta \leq 1$, measures the discrepancy between period 1 and period 2 preferences. Without loss of generality, we assume that the exponential-discounting factor δ equals 1. All results are qualitatively similar for non-trivial exponential discounting $\delta < 1$.[1]

If choices concerning risk and effort can be determined in the first period, a risk-averse agent eliminates any risk by selecting $a = 0$, and hence the optimal effort e_1 satisfies

$$u'(e_1) = c. \tag{3}$$

However, when the effort choice is made in the second period, after $a = 0$ has already been selected, the optimal effort e_2 solves

$$\beta u'(e_2) = c. \tag{4}$$

Hyperbolic-discounting preferences are time-inconsistent in the sense that the optimal level of effort depends on the period in which the decision is taken. In our case, time-inconsistency leads to sub-optimally low effort: $e_2 < e_1$ for any $\beta < 1$. The effort-provision problem becomes more severe, that is, e_2 decreases and the difference between e_1 and e_2 increases, when β goes down. When the agent cannot directly commit herself to e_1, she may be willing to influence her choice of effort by selecting a strictly positive exposure to risk, $a > 0$, in the first period.

To begin with, we would like to know when commitment through choosing a positive risk exposure is feasible. The following lemma shows that when the marginal utility is convex, a higher risk increases the agent's effort.

Lemma 1. *Let $e(a)$ be the optimal second-period effort. If $u'''(\cdot) > 0$, then $\frac{de(a)}{da} > 0$.*

Proof. Let $e(a)$ satisfy $\beta Eu'(e(a) + a\tilde{z}) = c$. Taking the derivative of this expression with respect to a, it yields

$$\frac{de(a)}{da} = -\frac{E\left(\tilde{z}u''(e(a) + a\tilde{z})\right)}{Eu''(e(a) + a\tilde{z})}, \tag{5}$$

which is positive when $u'''(\cdot) > 0$ by the covariance rule. □

This result is an illustration of the well-known property that risk-averse agents may work more when faced with a higher risk (e.g. Block and Heineke, 1973). The intuition behind the result is similar to that of precautionary saving under prudence (Kimball, 1990), with absolute prudence defined as $AP \equiv -u'''(\cdot)/u''(\cdot)$. Importantly, Lemma 1 however does not imply that the agent is willing to take more risk in the first place.

Deliberate exposure to risk may be an optimal strategy for time-inconsistent agents. To see this, note that the decision problem that the agent solves in period 1 takes the following form:

$$\max_a -ce + Eu(e + a\tilde{z}), \tag{6}$$

subject to

$$e = \arg\max_{\hat{e}} -c\hat{e} + \beta Eu(\hat{e} + a\tilde{z}). \tag{7}$$

Substituting (7) into (6) and differentiating with respect to a, the first and second derivatives of (6) are given by

$$-c\frac{de}{da} + E\left(u'(\cdot)\left(\frac{de}{da} + \tilde{z}\right)\right), \tag{8}$$

and

$$-c\frac{d^2e}{da^2} + E\left(u''(\cdot)\left(\frac{de}{da} + \tilde{z}\right)^2 + u'(\cdot)\frac{d^2e}{da^2}\right), \tag{9}$$

where

$$\frac{d^2e}{da^2} = -\frac{E\left(\tilde{z}u'''(\cdot)\left(\frac{de}{da} + \tilde{z}\right)\right)E\left(u''(\cdot)\right) - E\left(u'''(\cdot)\right)E\left(\tilde{z}u''(\cdot)\right)}{(Eu''(\cdot))^2}, \tag{10}$$

with all derivatives of $u(\cdot)$ having the same argument of $e + a\tilde{z}$. This allows us to formulate our main result.

Proposition 1. *Let a^* be the optimal choice of a of the problem (6) and (7) to e_2 as defined in (4). If*

$$-\frac{u'''(e_2)}{u''(e_2)}(1 - \beta) > -\frac{u''(e_2)}{u'(e_2)}, \tag{11}$$

or, equivalently, $AP(1 - \beta) > AR$, then $a^* > 0$.

Proof. Evaluated at $a = 0$, the first derivative (8) is zero and (10) becomes

$$\left.\frac{d^2e}{da^2}\right|_{a=0} = -\frac{u'''(e_2)}{u''(e_2)}. \tag{12}$$

Given (12), the second derivative (9) evaluated at $a = 0$ becomes

$$(c - u'(e_2))\frac{u'''(e_2)}{u''(e_2)} + u''(e_2), \tag{13}$$

which can be shown to be positive when condition (11) holds by using the definition of e_2. Since the first derivative is zero and the second derivative is positive, $a = 0$ is a local minimum and hence cannot be the solution of the problem (6) to (7). □

[1] In the standard δ–β model first used in Laibson (1997), these utility functions are $-ce + \delta Eu(\cdot)$, and $-ce + \beta\delta Eu(\cdot)$, respectively.

The above proposition implies that deliberate risk exposure $a > 0$ is more likely when time-inconsistency and absolute prudence (AP) increase and when absolute risk aversion (AR) decreases. The intuition behind condition (11) is as follows. The time-inconsistency effect $1-\beta$ measures the degree of the effort-provision problem, i.e. when $1-\beta$ increases, effort goes down. Absolute prudence—see expression (12)—is related to the reaction of effort to a change in the amount of risk and, therefore, measures the efficiency of risk exposure as an intrapersonal commitment device for exerting a higher effort. In contrast, absolute risk aversion reflects the negative consequences of risk and captures the cost of deliberate exposure to risk. The agent becomes more inclined to bear this cost as the effort-provision problem becomes more severe.

Finally, positive risk exposure in the optimum is closely related to the property of decreasing absolute risk aversion:

Corollary. *A necessary condition for positive risk exposure ($a^* > 0$) to be optimal is*

$$-\frac{u'''(e)}{u''(e)} > -\frac{u''(e)}{u'(e)},$$

i.e. optimal positive risk exposure obtains whenever $u(\cdot)$ satisfies the property of decreasing absolute risk aversion (DARA) at e.

3. Discussion and conclusion

Our model can be reinterpreted as one in which an agent selects, in the first period, among N different projects with identical expected benefits but different levels of risk dispersion: $0 \le a_1 < a_2 < \cdots < a_N$. For a risk-averse time-consistent agent, a_1 is clearly the optimal choice. In contrast, we have shown that a hyperbolic-discounting agent takes into account two effects of risk: losses due to risk aversion and benefits in terms of higher effort due to the value of commitment. If the second effect is sufficiently large relative to the first, then some $a^* > a_1$ is optimal. More generally, if the choice is between projects that differ not only in risk but also in expected benefits (i.e. relaxing the zero-mean assumption on $\tilde{z}$), following a similar line of reasoning it can be shown that a time-inconsistent decision maker may select the project with a higher risk compared to the one selected by a perfectly time-consistent agent.

The possibility of commitment by exposure to risk is also not restricted to the case of a risky project but can be applied in a number of other settings such as investments in risky assets. When the uncertainty about the outcome of own effort is fixed and cannot be modified (e.g. because of a fixed wage schedule), the agent may still use this commitment mechanism if she receives her income from different sources. Before selecting her effort, assume that she can allocate her initial wealth, normalized to W, between a risky and a risk-free asset which have the same expected return normalized to zero. Investing $a \le W$ in the risky asset yields a benefit of $a\tilde{z}$, where $\tilde{z}$ is such that $E(\tilde{z}) = 0, E(\tilde{z}^2) = 1$. Hence the total income is given by $e + a\tilde{z}$, which is consistent with our previous analysis.

In summary, the type of commitment identified in this note includes not only situations in which agents choose between different assets or projects, but comprises any lottery or investment decisions that increase the volatility of the agent's total income. Even though behavior of this kind may seem irrational for risk-averse agents (or may seem to imply risk-loving preferences), time-inconsistency provides a novel rationale for deliberate exposure to risk under prudence. These considerations should also be taken into account when risk preferences are inferred from observed choices, e.g. in empirical work.

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