Hindsight-biased evaluation of political decision makers ☆

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ABSTRACT

Hindsight bias is a cognitive deficiency that leads people to overestimate ex post how predictable an event was. In this paper we develop a political-agency model in which voters are hindsight-biased and politicians differ in ability, defined as information concerning the optimal policy. When public information is not too accurate, low-ability politicians sometimes gamble on suboptimal policies: in an attempt to mimic the high-ability type, who has superior private information, they go against public information and choose a policy whose expected payoff to society is negative. We model hindsight bias as a memory imperfection that prevents voters from accessing their ex ante information about the state of the world. We show that the bias can act as a discipline device that reduces policy gambles and can therefore be welfare enhancing. Although it is well known that restrictions on information acquisition can be beneficial for a principal, our contribution is to show that a psychological bias can have such an effect.

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1. Introduction

The psychology literature provides ample evidence that many people exhibit hindsight bias: in retrospect, they systematically over-estimate how predictable an event was.[1] The bias can cause problems when a decision maker is evaluated after the outcome of his actions is known, and no ex ante contract, mapping outcomes into performance assessments, exists. Democratic elections are a prominent example of such a situation. Voters typically have to make an ex post assessment of

the decisions of an incumbent politician and decide whether to confirm him in office or replace him with a challenger. Hindsight bias is often thought to adversely affect the voters' decision by altering their perception of the incumbent's responsibility (Camerer et al., 1989; Frey and Eichenberger, 1991). Because they deem observed outcomes more predictable than they actually were, hindsight-biased voters tend to give the incumbent less credit than is due for success and more blame than is warranted for failure.

In this paper we argue that hindsight bias may not be altogether detrimental to the functioning of the political system because it can have a disciplining effect on politicians. We present a model in which the incumbent politician has a tendency to engage in policy gambles: to mislead voters into thinking that he has superior private information, the incumbent goes against public information and chooses a policy whose expected payoff to society is negative but which, in the event of success, boosts his chances of staying in office. We show that hindsight bias on the part of voters can reduce such policy gambles and can therefore be welfare-enhancing.

To motivate our analysis, let us briefly consider two episodes from U.S. politics: the first Gulf war, and the Iran hostage crisis. The Gulf war of 1991 is a well-documented example of how hindsight bias can influence the public perception of policy choices, and arguably the outcome of elections. President George H.W. Bush had initiated military action in response to the Iraqi invasion of Kuwait, an endeavor of considerable political risk, and came away with what observers unanimously viewed

as a huge success. Opinion polls about the decision to go to war clearly bear out voters' hindsight bias:

> In December 1990, respondents had split about 50/50 on a question asking whether they preferred sanctions or military action. But when asked after the war how they had felt before the war, those inclined to remember that they had supported military action outnumbered those recalling their support for sanctions by nearly four to one. (Mueller, 1994, p. 87)

If voters considered the decision to go to war as an easy call in retrospect, such a belief would surely have been detrimental to the President's hope of winning reelection.[2]

The Iran hostage crisis illustrates the fact that policy gambles are an important political accountability problem. In 1979, 66 U.S. diplomats had been taken hostage in the Tehran embassy. President Jimmy Carter's low approval ratings and a slumping U.S. economy meant that the president was unlikely to be confirmed in office unless he somehow managed to resolve the crisis before the November 1980 election. As negotiations with Iran dragged on, Carter started to contemplate a military solution that most observers at the time thought impossible or too risky. Polls showed that only 10% of voters were in favor of using military force (Brulé, 2005), and Secretary of State Cyrus Vance resigned because of disagreement with Carter and his team of advisors on the issue (Houghton, 2001). In spite of the opposition, Carter decided to launch a high-risk military rescue operation. It became one of the best-known fiascos in the history of American military intervention.[3]

How does hindsight bias reduce the tendency toward policy gambles? The reason why a politician may sometimes choose a policy that both he and the public expect to reduce welfare is that, if the gamble pays off, rational voters are "surprised" and make an upward adjustment of their beliefs about the politician's ability. Hindsight-biased voters, however, think they knew all along that the policy was going to work (they are not surprised), and do not give him as much credit. Therefore, when facing hindsight-biased voters, the politician is less tempted to gamble on an inefficient policy in the first place.

To be more precise, in the model we develop, a politician whose ability is unknown to voters has to choose between two policies. While voters and low-ability politicians obtain only an imperfect signal about which policy is preferable ex ante, high-ability politicians know the state of the world with certainty. This setup creates incentives for a low-ability politician to inefficiently ignore public information concerning the welfare-maximizing policy in an attempt to "look smart," i.e., to make it seem as if he had superior private information, the trademark of high-ability politicians. To see why, consider what happens in the case of rational voters, noting that a high-ability politician always chooses the right policy and thus disregards the publicly observed signal. If the low-ability politician always follows the signal, rational voters infer that any politician who chooses a policy that is contrary to the signal must be of high ability; thus, successfully implementing an unpopular policy signals competence.

We show that if the public signal is in an intermediate range such that neither policy choice clearly dominates the other in terms of expected welfare, the equilibrium with rational voters has the low-ability politician randomizing between choosing the policy that is optimal according to the signal and doing exactly the opposite. This randomization is detrimental to welfare because policy choices do not use all available information.

We assume that voters suffering from hindsight bias distort their recollection of the signal so as to make it consistent with the state of the world revealed by the policy outcome. If the signal suggested that the state of the world is 0, but the politician successfully enacts a policy that reveals that the opposite is the case, then voters wrongly believe that the signal had suggested all along that the state was 1. Therefore, with hindsight-biased voters, some of the gain in reputation that follows from an unpopular policy which turns out to be a success is destroyed. Ex post, biased voters think that it was the obvious choice anyway. Anticipating voters' biased belief updating, the low-ability politician gambles on the inefficient policy less often when voters are hindsight biased than when they are rational. Thus, hindsight bias on the part of voters can act as a discipline device.

Another way to interpret the effect of hindsight bias is the following. In our model, politicians sometimes try to improve their reputation by going against the public signal. Hindsight bias makes voters forget the public signal and replace it with an erroneous recollection based on outcome knowledge. By doing so, the bias takes away the tool of going against the public signal.

We also show that when the two policies are asymmetric with respect to the probability of revealing the state of the world, hindsight bias may have a less benign effect. Because revelation of the state of the world has the potential to uncover the incumbent's lack of ability, the low type may refrain from choosing the more revealing policy even though it is efficient. Under hindsight-biased evaluation, the politician does not get as much credit for success as he deserves. Compared to rational evaluation, he may be more inclined to choose the policy that has lower expected welfare but whose outcome is certain and does not entail the risk of uncovering him as a low type. Thus, hindsight bias can sometimes discourage efficient risk taking.

Even when the disciplining effect prevails, so that hindsight bias increases voters' first-period welfare, an overall welfare assessment also has to take into account the second (i.e., post-election) period. We discuss how hindsight bias affects the selection of the second-period politician and conclude that the impact of the bias is ambiguous. On the one hand, biased voters hold erroneous posteriors about the incumbent's ability, sometimes leading them to elect the wrong candidate. On the other hand, hindsight bias may generate offsetting benefits in terms of inferences about the politician's type. By changing the low type's behavior in the first period, the bias can make it easier for voters to distinguish low- from high-ability politicians.

Our contribution to the literature is twofold. First, our result that a behavioral bias can improve welfare adds to the behavioral-economics literature. It is similar in spirit to Compte and Postlewaite (2004), Bénabou and Tirole (2002) and Köszegi (2006). Those papers, however, consider how a psychological bias (namely, overconfidence) affects *intrapersonal* welfare.[4] By contrast, we investigate how a bias on the part of one group of agents (voters) can affect the behavior of other agents (politicians) in a way that increases the welfare of the former. We use a standard welfare measure that is unaffected by which "self" of an individual one considers; nor does it involve belief consumption.

Second, our paper extends the literature on political agency, in particular by going beyond the standard rational-voter assumption, as suggested by Besley (2006). Our basic model is related to the recent literature on the dysfunctional effects of reelection concerns which can arise when politicians have better information than voters. This literature has mainly emphasized distortions such as inefficient policy experimentation and pandering (see, e.g., Majumdar and Mukand, 2004; Maskin and Tirole, 2004; Smart and Sturm, 2006). The

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[2] We discuss this example in more detail in Section 5.

[3] A phenomenon that arises as a special case of the policy gambles in our model is gambling for resurrection. Downs and Rocke (1994) provide historical examples of gambling for resurrection by policy makers. Systematic evidence is difficult to come by because the information needed to identify a behavior as gambling for resurrection is private. Hess and Orphanide's (1995) finding that U.S. presidents are more likely to start a war in an election year when the economy is doing badly nevertheless suggests that such behavior is not merely anecdotal.

[4] In Compte and Postlewaite (2004), an agent's self-confidence affects his performance in a task. Information-processing biases such as repressing memories of bad performance can improve the individual's welfare by boosting his confidence, thus helping him do better. Bénabou and Tirole (2002) show how overconfidence can help an individual overcome time inconsistency and thus improve his well-being (at least from an ex ante ("self zero") perspective). Köszegi (2006) lets individuals consume their self-perception, so that an overly positive self-image can raise utility.

inefficiency in our model consists in politicians signaling their ability by choosing an *unpopular* policy.

Canes-Wrone et al. (2001) obtain both kinds of inefficient behavior in a model where politicians try to signal their ability and the policies available differ in the probability of uncertainty resolution, i.e., the probability that the policy outcome is realized before the election. In contrast to our model, the quality of the challenger is known at the outset. When the challenger's quality is low, the low-ability incumbent may choose a suboptimal policy simply because it is popular among voters (pandering). When the challenger's ability is high and the probability of uncertainty resolution is sufficiently asymmetric, however, the incumbent may engage in something the authors call "fake leadership": he acts against both popular belief and his private signal, trying to be perceived as a leader.[5] In our model, inefficient policy gambles (i.e., fake leadership) occur regardless of the asymmetry between probabilities of uncertainty resolution, although the effect of hindsight bias depends on it.

Our results are similar to those obtained by Levy (2004) for the case of decision makers with career concerns. The decision makers in her model display a behavior labeled "anti-herding," i.e., they have a tendency to take decisions contradicting the public prior. In Prat (2005), the agent may disregard valuable private information in an attempt to mimic the more able type. As a result, the principal may be better off not observing the agent's action.

The finding that less information can be better for a principal is a common theme in setups where the principal lacks commitment power. It is a feature of Crémer (1995), for example, where the principal may forego a costless monitoring technology in order to increase the agent's incentives. In our model, incentives for the low type are improved due to voters' distorted memories; it is their hindsight bias that destroys information. Although it is well known that restrictions on information acquisition can be beneficial to the principal, our contribution is to show that a psychological bias can have such an effect.

The remainder of the paper is organized as follows. Section 2 introduces the model and defines a hindsight-biased information structure. Section 3 determines the equilibria under rational and hindsight-biased policy evaluation and compares their welfare properties. Section 4 discusses our modeling of hindsight bias and outlines the effect of the bias on selection. Finally, Section 5 concludes. All proofs are relegated to the Appendix.

2. Model

We consider a simple two-period political agency model. In each period, the state of the world ω can be either 0 or 1. Politicians and voters receive an imperfect public signal $\sigma \in \{\sigma_0, \sigma_1\}$ about the state of the world. Let $\nu \in \{\nu_0, \nu_1\}$ denote the probability that the state of the world is 1 conditional on signal σ. That is, $\nu_0 \equiv Pr[\omega = 1|\sigma_0]$ and $\nu_1 \equiv Pr[\omega = 1|\sigma_1]$. There are two possible policies (or actions) $a \in \{a_0, a_1\}$ from which the decision maker can choose. The actions can produce different policy outcomes $y \in \{0, \kappa, 1\}$ (defined as the payoff to voters), which depend on the state of the world. Policy a_1 delivers a payoff of 1 if $\omega = 1$, and 0 if $\omega = 0$. Thus, the outcome associated with a_1 is always contingent on ω. The outcome associated with policy a_0 is contingent on ω only with probability p, in which case it delivers a payoff of 1 if $\omega = 0$, and 0 if $\omega = 1$. With probability $1 - p$, policy a_0 delivers a payoff of κ, independent of ω. We assume $0 < \kappa < 1$ so that a_1 yields a higher payoff than a_0 if and only if $\omega = 1$.

A politician can be of high or low ability and knows his own type $\theta \in \{\theta_H, \theta_L\}$. The prior probability $\lambda \in (0, 1)$ that the incumbent is of high ability is common knowledge. High ability is defined as an informational advantage over voters as to the welfare-maximizing policy. While a low-ability politician only learns the signal (σ), a high-ability politician also learns the state of the world (ω).

Our assumption that voters obtain an informative signal about the state of the world, just as politicians do, and that the signal coincides with that of low-ability politicians, reflects the idea that voters are exposed to a certain amount of policy-relevant public information (e.g., from the media). On average, politicians still have "expertise," i.e., they are more likely to have correct information concerning the underlying state of the world than voters.

We impose the following assumption on the signal:

Assumption 1. $0 < \nu_0 < 1/2 < \nu_1 < 1$.

This assumption ensures that the public signal is informative but imperfect: while it does not reveal the state of the world with certainty, the likelihood that the state is ω after signal σ_ω is greater than the likelihood that the state is $1 - \omega$. In other words, the signal is more likely to be correct than incorrect.

In order to model hindsight bias, we introduce a distinction between the original signal σ received by all players before the politician chooses an action, and the *recollection* $\hat{\sigma}$ of the original signal that voters have when deciding who to vote for. While rational voters always correctly recall the signal ($\hat{\sigma} = \sigma$), hindsight-biased voters' recollection sometimes differs from the original signal, as we outline below.

2.1. Timing

The game is played in two periods (interpreted as terms in office). Period 1 is divided into four stages; see Fig. 1. At $t = 0$, nature draws the incumbent's type θ, the state of the world ω, and the public signal σ. All types of incumbents and voters observe σ but only type-θ_H incumbents learn ω. At $t = 1$, the incumbent decides which policy to implement. At $t = 2$, the outcome of the policy is realized and learned by all players. At $t = 3$, the election stage of the game, voters choose between the incumbent and a challenger. The electorate's perception of the challenger (i.e., the probability that the challenger is of high ability) is λ_c, which is randomly drawn from a distribution on $[0, 1]$. The perception of the incumbent's ability depends on the recollection $\hat{\sigma}$ that voters have of the original signal σ at this stage. In the second period, following a draw of ω and σ, the appointed politician takes an action. After that, the second-period outcome is realized and publicly observed. At this point, the game ends.

2.2. Politicians and voters

The voters' task is to decide whether to reelect or replace the incumbent. Their strategy consists of a probability distribution over the actions "reelect the incumbent" and "elect the challenger" for each possible combination of recalled signal, action, outcome, and perceived ability of the challenger. The voters' payoff equals the expected social welfare.

The politician's preferences are given by

$$u = \phi W + (1 - \phi) Pr[\text{reelection}],$$

where W is current-period social welfare.[6] The parameter $\phi \in (0, 1)$ measures the politician's relative concern for welfare and reelection. We assume that it satisfies the following assumption:

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[5] Canes-Wrone et al. (2001) distinguish fake leadership from true leadership, the latter being defined as a high-ability incumbent ignoring public information and following his superior private signal, a behavior that is in the public interest.

[6] The assumption that all types of politician care about social welfare, as well as holding office, can be justified, for example, by the fact that politicians are drawn from the population of voters. Thus, they can be expected to consume the same goods as the rest of the electorate. This formulation follows authors such as Rogoff (1990), Harrington (1993), Canes-Wrone et al. (2001) and Majumdar and Mukand (2004).
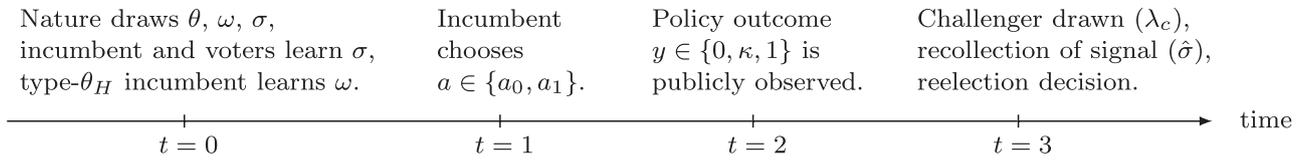
Nature draws θ, ω, σ, incumbent and voters learn σ, type-θ_H incumbent learns ω.	Incumbent chooses $a \in \{a_0, a_1\}$.	Policy outcome $y \in \{0, \kappa, 1\}$ is publicly observed.	Challenger drawn (λ_c), recollection of signal ($\hat{\sigma}$), reelection decision.	
$t = 0$	$t = 1$	$t = 2$	$t = 3$	time

Fig. 1. Timing of the game (period 1).

Assumption 2. $(1-\phi)/\phi < min\{1, (p+(1-p)\kappa)/\lambda, (1-(1-p)\kappa)/\lambda\}$.

This assumption puts an upper bound on the strength of reelection concerns. As we show below, it rules out equilibria in which the high-ability politician signals his ability by deliberately producing a failure. It also rules out pooling equilibria.[7]

In the second period, there are no reelection concerns, so politicians' and voters' objectives are perfectly aligned. All types of politicians try to maximize welfare, but they are not equally good at it. The voters' optimal strategy is therefore to elect the candidate they perceive as more competent. Let $\mu(\hat{\sigma}, a, y)$ denote the voters' posterior belief that the incumbent is of type θ_H given recalled signal $\hat{\sigma}$, policy choice a and realized outcome y. We refer to belief μ as the incumbent's reputation. Voters reelect the incumbent if and only if $\lambda_c \leq \mu(\hat{\sigma}, a, y)$. For simplicity, we assume that λ_c is uniformly distributed on [0,1]. The incumbent's probability of reelection thus equals his reputation.

The politician's payoff is his expected utility U. Let α denote a mixed action such that the politician plays a_1 with probability α and a_0 with probability $1 - \alpha$. The expected utility given the voters' behavior and the information available to the politician is $U(\alpha, \mu, \Psi_\theta) = \alpha[\phi E(y|a_1, \Psi_\theta) + (1-\phi)E(\mu|a_1, \Psi_\theta)] + (1-\alpha)[\phi E(y|a_0, \Psi_\theta) + (1-\phi)E(\mu|a_0, \Psi_\theta)]$, where Ψ_θ is the politician's (type dependent) information set:

$$\Psi_\theta = \begin{cases} (\omega, \sigma) & \text{for} \quad \theta = \theta_H \\ \sigma & \text{for} \quad \theta = \theta_L. \end{cases}$$

A politician's strategy prescribes a probability $s(\theta, \Psi_\theta)$ of playing a_1 for each type θ and each possible realization of Ψ_θ.[8] We will refer to a set of strategies and beliefs as an equilibrium if (*i*) strategies are optimal given beliefs, i.e., $\forall \theta, \forall \Psi_\theta$, $s(\theta, \Psi_\theta) \in arg\,max_\alpha \, U(\alpha, \mu, \Psi_\theta)$, and voters reelect the incumbent if and only if $\lambda_c \leq \mu(\hat{\sigma}, a, y)$, (*ii*) beliefs are derived from equilibrium strategies and observed actions using Bayes' rule whenever possible, i.e.,

$$\mu(\hat{\sigma}, a, y) = \frac{\lambda Pr(\hat{\sigma}, a, y | \theta_H)}{\lambda Pr(\hat{\sigma}, a, y | \theta_H) + (1-\lambda)Pr(\hat{\sigma}, a, y | \theta_L)},$$

and (*iii*) voters hold pessimistic out-of-equilibrium beliefs, i.e., $\mu(\hat{\sigma}, a, y) = 0$ for any triplet $(\hat{\sigma}, a, y)$ that is off the equilibrium path. The equilibrium concept defined by conditions (*i*) through (*iii*) is a refined version of perfect Bayesian equilibrium (PBE) allowing for memory distortions on the part of voters. In what follows we drop the politician's type from the specification of strategies as this cannot lead to confusion. Thus, we write $s(\omega, \sigma)$ for $s(\theta_H, (\omega, \sigma))$ and $s(\sigma)$ for $s(\theta_L, \sigma)$.

2.3. Hindsight bias

Evaluators are hindsight biased if, as Rabin (1998, p. 30) puts it, they "exaggerate the degree to which their beliefs before an informative event would be similar to their current beliefs." In our model, the informative

event is given by the revelation of the policy outcome y. Voters' beliefs about ω before the informative event are determined by the signal σ, while voters' beliefs about ω after the informative event are determined by which state of the world is revealed by the outcome y (if any). Following research in psychology, we interpret hindsight bias as a memory imperfection according to which an evaluator follows some mental strategy to *reconstruct* the original prior from the default information he holds ex post; this is what Hawkins and Hastie (1990, p. 321) call "reconstruction of the prior judgment by 'rejudging' the outcome."[9] The following definition of hindsight bias ensures consistency of the set of recalled prior beliefs with the set of original prior beliefs an evaluator may hold about the state of the world.[10]

Definition (Hindsight bias with a binary signal). After an informative policy outcome, $y \in \{0, 1\}$ voters overestimate the accuracy of their prior belief about the state of the world. Let ω be the true state of the world, as revealed by y. Irrespective of the original signal, the voters' recalled prior is based on σ_ω.

Table 1 specifies which signal hindsight-biased voters will recall, according to this definition, as a function of policy choice and outcome. If $\sigma = \sigma_0$ and voters learn that $\omega = 1$, they think that ex ante they attached probability ν_1 to the state of the world being 1 even though according to their original signal it was $\nu_0 < \nu_1$. Thus, they erroneously recall $\hat{\sigma} = \sigma_1$ rather than σ_0. Similarly, if $\sigma = \sigma_1$ and they learn that $\omega = 0$, hindsight-biased voters think that ex ante they attached probability $1 - \nu_0$ to the state of the world being 0 even though according to their original signal it was $1 - \nu_1 < 1 - \nu_0$; they recall $\hat{\sigma} = \sigma_0$ rather than σ_1. While outcomes $y = 0$ and $y = 1$ reveal the state of the world with certainty, outcome $y = \kappa$ is uninformative about ω. The recollection of prior probabilities is only altered when new information about the state of the world is revealed. Therefore, the recalled signal is not distorted after $y = \kappa$. To close the model, we make an assumption about the voters' awareness of their bias:

Assumption 3. Hindsight-biased voters are unaware of their memory imperfection: they believe that the signal they recall is the true signal.

According to this assumption, hindsight-biased voters are *naive* in the sense that they are certain that they correctly remember the signal even though the bias sometimes causes erroneous recollection. The assumption of naivety simplifies the exposition without affecting the qualitative results, as we discuss in Section 4.[11]

3. Hindsight bias as a discipline device

In this section, we derive the equilibrium of the game. We compare rational evaluation with hindsight-biased evaluation and study the

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[7] In a pooling equilibrium, all types of politician choose the same policy irrespective of their information and voters believe that any politician who deviates is of low ability. Pooling equilibria could also be pruned by using a refinement (namely, the D1 criterion), in which case Assumption 2 could be reduced to $(1-\phi)/\phi < 1$.

[8] Strictly speaking, there are *three* types of politicians in this model: two types of high-ability politicians (one for each realization of the first-period state of the world), and one type of low-ability politician. We have labeled types in more intuitive terms for greater clarity of exposition.

[9] Following a similar cognitive view as in our model, Hoffrage et al. (2000) emphasize the fact that for individuals with capacity-constrained memory, holding current information in memory is, for general tasks, more important and accurate than remembering past prior probabilities.

[10] For a continuous formulation of a closely related bias known as "curse of knowledge," see Camerer et al. (1989). They derive the bias from a violation of the law of iterated expectations in which the recalled prior belief about ω is located somewhere between the true prior and the posterior probability. Although similar in logic, we do not follow their formulation as it would lead to a contradiction with the set of possible priors in a binary model.

[11] For a memory-based model of bounded rationality see Mullainathan (2002); there, as in our model, a decision maker takes the recalled history of signals as the true history.

Table 1
Biased recollection $\hat{\sigma}$ of original signal σ.

Outcome	Policy choice	
	a_0	a_1
$y = 0$	σ_1	σ_0
$y = \kappa$	σ	–
$y = 1$	σ_0	σ_1

effect of the bias on first-period welfare. As a benchmark, we look at the welfare-maximizing policy choice for the politician. The high-ability politician should always choose the policy corresponding to the state of the world. The low-ability politician should choose policy a_0 if and only if $p(1-\nu) + (1-p)\kappa \geq \nu$, defining a threshold value of ν given by

$$\nu^* \equiv \frac{p + (1-p)\kappa}{1 + p} \tag{1}$$

such that policy a_0 is optimal for $\nu \leq \nu^*$ and policy a_1 is optimal for $\nu > \nu^*$. Thus, whatever the realization of the signal and the values of ν_0 and ν_1, there is always a unique policy that the low-ability politician should choose in order to maximize expected welfare given the available information.

Turning to the analysis of equilibrium, we begin by establishing that Assumption 2 rules out equilibria in which the high-ability politician does not implement the welfare-maximizing policy.

Lemma 1. Under Assumption 2, there cannot be an equilibrium in which the high-ability politician deviates from the welfare-maximizing policy.

Lemma 1 implies that, if an equilibrium exists, it must be such that the high-ability politician implements the welfare-maximizing policy, that is, he chooses the policy corresponding to the state of nature. Formally, $s(0,\sigma) = 0$ and $s(1,\sigma) = 1$, for any σ. When reelection concerns are not too strong, the high-ability politician never plays a strategy that requires him to choose a socially suboptimal policy. The reason, as shown in the proof, is that the high type always has stronger incentives to deviate from a suboptimal to the optimal policy than the low type. Note that the result of the Lemma does not depend on whether voters are rational or hindsight-biased; as long as Assumption 2 holds it is true for any beliefs. Accordingly, suppose voters believe that type θ_H plays the action corresponding to the state of the world. Under Assumption 3, voters' beliefs are then given by

$$\mu(\hat{\sigma}, a, 0) = 0 \ \forall \hat{\sigma}, a \tag{2}$$

$$\mu(\hat{\sigma}, a_0, \kappa) = \mu(\sigma, a_0, \kappa) = \frac{\lambda(1-\nu)}{\lambda(1-\nu) + (1-\lambda)(1-s(\sigma))}$$

$$\mu(\hat{\sigma}, a_0, 1) = \frac{\lambda}{\lambda + (1-\lambda)(1-s(\hat{\sigma}))}$$

$$\mu(\hat{\sigma}, a_1, 1) = \frac{\lambda}{\lambda + (1-\lambda)s(\hat{\sigma})}.$$

Note that these expressions depend on the voters' *recalled* signal $\hat{\sigma}$, which does not always coincide with the original signal σ under hindsight-biased evaluation (except when $y = \kappa$, in which case σ is always correctly remembered). A low-ability politician's expected payoff from a_0 is then

$$\phi[(1-p)\kappa + p(1-\nu)] + (1-\phi)[(1-p)\mu(\hat{\sigma}, a_0, \kappa) + p(1-\nu)\mu(\hat{\sigma}, a_0, 1)], \tag{3}$$

while from a_1 it is

$$\phi\nu + (1-\phi)\nu\mu(\hat{\sigma}, a_1, 1). \tag{4}$$

We will categorize equilibria according to the low type's equilibrium strategy: when $s(\sigma) = 0$ or $s(\sigma) = 1$, we speak of a pure-strategy equilibrium, while for $0 < s(\sigma) < 1$ (the low type randomizes between a_0 and a_1) we speak of a mixed-strategy equilibrium.[12]

3.1. Equilibrium with rational voters

Rational voters always correctly recall the original signal, so $\hat{\sigma} = \sigma$. Using this to replace μ according to Eq. (2) in expressions (3) and (4), the low-ability politician's payoff from action a_0 becomes

$$\phi[(1-p)\kappa + p(1-\nu)] + (1-\phi)\left[\frac{(1-p)\lambda(1-\nu)}{\lambda(1-\nu) + (1-\lambda)(1-s(\sigma))} + \frac{p(1-\nu)\lambda}{\lambda + (1-\lambda)(1-s(\sigma))}\right], \tag{5}$$

while his payoff from a_1 becomes

$$\phi\nu + (1-\phi)\frac{\nu\lambda}{\lambda + (1-\lambda)s(\sigma)}. \tag{6}$$

Because the politician cares about his reputation as well as welfare, his incentives are not perfectly aligned with those of voters. The following lemma describes the equilibrium of the resulting game with rational evaluation, for which $s^R(\sigma)$ denotes the low type's equilibrium strategy.

Lemma 2. Suppose voters are rational. There exists a unique equilibrium characterized by thresholds $\underline{\nu}^R$ and $\overline{\nu}^R$, such that the high-ability politician chooses a_0 when $\omega = 0$ and a_1 when $\omega = 1$, while for $\nu \leq \underline{\nu}^R$ the low-ability politician always plays a_0 ($s^R(\sigma) = 0$), for $\nu \geq \overline{\nu}^R$ he always plays a_1 ($s^R(\sigma) = 1$), and for $\underline{\nu}^R < \nu < \overline{\nu}^R$ he randomizes between a_0 and a_1: $s^R(\sigma) = S^R(\nu)$ where $S^R(\nu) \in (0,1)$ is increasing in ν.

In the unique equilibrium, the high-ability politician always chooses the socially optimal policy. The low-ability politician's equilibrium behavior depends on the probability that $\omega = 1$, given by ν. When ν is small, we are in a pure-strategy equilibrium in which the low-ability politician always chooses a_0. When ν is large, we are in a pure-strategy equilibrium in which he always chooses a_1. When ν is in an intermediate range, we are in a mixed-strategy equilibrium in which he randomizes and chooses each action with probability $S^R(\nu)$, obtained by equating Eqs. (5) and (6).

Note that Lemma 2 describes the equilibrium for both the σ_0 and the σ_1 subgame. For example, suppose $\nu_0 < \underline{\nu}^R < \nu_1 < \overline{\nu}^R$. Then, $s^R(\sigma_0) = 0$ and $0 < s^R(\sigma_1) < 1$, i.e., the low type plays a_0 with certainty after receiving signal σ_0, and randomizes after receiving signal σ_1. Together with Assumption 1, Lemma 2 implies that $s^R(\sigma_0) \leq s^R(\sigma_1)$, i.e., the low-ability politician is more likely to play a_1 after σ_1 than after σ_0. It follows immediately from the fact that there can be a mixed-strategy equilibrium that the low type's behavior exhibits inefficiencies, as optimality requires the low type to follow the cutoff rule of playing a_0 if and only if $\nu \leq \nu^*$, and a_1 otherwise (formally, $\underline{\nu}^R = \overline{\nu}^R = \nu^*$). The following proposition makes the nature of the inefficiencies more precise.

Proposition 1. The equilibrium with rational voters involves inefficient behavior by the low-ability politician for some part of the signal space: $\overline{\nu}^R > \nu^*$, and $\underline{\nu}^R < \nu^*$ iff $\lambda < \overline{\lambda}$, where

$$\overline{\lambda} \equiv \frac{1 - \nu^*(1-\nu^*) - \sqrt{(1-\nu^*(1-\nu^*))^2 - 4(\nu^*)^2(1-\nu^*)p}}{2\nu^*(1-\nu^*)p}. \tag{7}$$

As $p \to 1$, $\overline{\lambda} \to 1$.

Proposition 1 tells us that there is always a range of ν for which the low-ability politician plays a_0 too often (i.e., he sometimes chooses a_0

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even though a_1 is optimal); moreover, if λ is not too large, there is also a range of ν for which the low-ability politician plays a_1 too often. The result is illustrated in Fig. 2 for the case where $\lambda < \overline{\lambda}$ and thus $\underline{\nu}^R < \nu^* < \overline{\nu}^R$. In this case, inefficiencies arise for all values of ν between $\underline{\nu}^R$ and $\overline{\nu}^R$.

The basic intuition is that, because the high-ability politician disregards the public signal and follows his superior private information, deviating from the policy choice that the public believes to be optimal can signal competence. While producing a failure ($y=0$) always leads to a reputation of zero, producing a success ($y=1$) has asymmetric effects on the politician's reputation depending on how likely the voters perceive a success as coming from a high type. When there is a policy that the low type is not supposed to choose in equilibrium, voters believe that anyone who successfully implements this policy must be of the high type. This creates incentives for the low-ability politician to ignore the information provided by the public signal and choose the opposite of what voters believe to be optimal.

To better understand the result, it is instructive to consider two special cases: $p=0$ and $p=1$. When $p=1$, the game is symmetric in the sense that both actions reveal the state of the world with probability 1. For values of ν that are in an intermediate range such that neither policy is clearly better than the other in terms of welfare, it cannot be an equilibrium for the low type to always choose the optimal policy. Suppose it were. Then, the low type could fool voters into thinking he is of the high type by successfully implementing the other policy, which has about equal probability of being successful but a much stronger effect on reputation. Neither can it be an equilibrium to always choose the suboptimal policy, since then the politician could improve both components of his utility (welfare and reputation) by deviating to the optimal policy. Thus, for ν close to 1/2, the only possible equilibrium has the low-ability politician randomizing between policies: sometimes he follows the signal and chooses the efficient policy, and sometimes he goes against the signal and gambles on the inefficient policy. Formally, with $p=1$ the cutoff values characterizing the low type's strategy are such that $0 < \underline{\nu}^R = 1 - \overline{\nu}^R < 1/2 = \nu^*$.

On the other hand, when $p=0$, the game is strongly asymmetric in that action a_0 never reveals the state of the world whereas a_1 always does. For large values of ν, the above argument still applies qualitatively: when a_1 is better than a_0 in terms of expected welfare, but not by too much, the equilibrium is in mixed strategies; the low type plays a_0 too often (i.e., $\overline{\nu}^R > \nu^*$). For lower values of ν, however, the low-ability politician's decision problem changes, as he can now insure himself against failure and the associated loss of reputation by playing a_0. This is particularly attractive when his initial reputation λ is high ($\lambda > \overline{\lambda}$), so that in the absence of new information he stands a good chance of being reelected. He will then refrain from gambling on a_1 for all $\nu < \nu^*$, and even choose a_0 with probability 1 for some values of $\nu > \nu^*$ (i.e., $\underline{\nu}^R > \nu^*$).
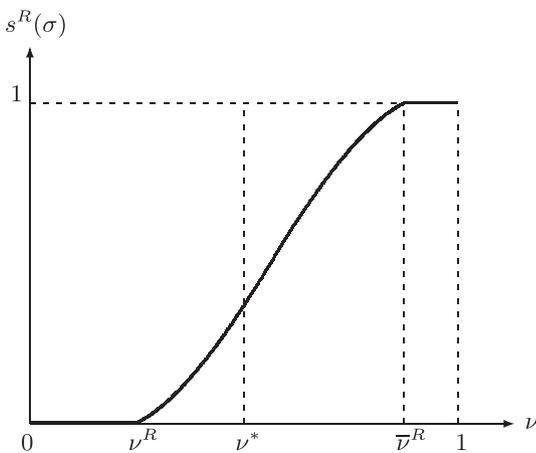

Fig. 2. The equilibrium strategy for type θ_L with rational voters.

By contrast, when λ is low ($\lambda < \overline{\lambda}$), we have a situation that can be interpreted as the incumbent being "behind in the polls." In that case, his chances of reelection are small unless he manages to change voters' opinion, which requires a political success. Thus, the low-ability politician sometimes gambles on the more revealing policy a_1 which, given $\nu < \nu^*$, has negative expected social value but gives him a better chance of being reelected (i.e., $\underline{\nu}^R < \nu^*$).

3.2. Equilibrium with hindsight biased voters

In solving for the equilibrium of the game with hindsight-biased voters, we assume that politicians anticipate voters' bias and that voters correctly predict the politician's equilibrium strategy. Thus, for each realization of the signal, voters know the probability that the equilibrium strategy assigns to the low-ability politician playing a_1. To calculate their posterior beliefs, however, voters use the probability that corresponds to the signal they recall. Because of hindsight bias, the signal voters recall may differ from the original signal, so their posterior beliefs can be wrong.

We now have to examine the subgames corresponding to the two signals separately. Consider a low-ability politician pondering his policy choice after receiving signal $\sigma = \sigma_0$. He knows that if he chooses policy a_0 and fails ($y=0$) or chooses a_1 and succeeds ($y=1$), voters will wrongly believe they "knew all along" that the state of the world was $\omega = 1$, i.e., their recalled signal will be $\hat{\sigma} = \sigma_1$. When forming their posterior about the incumbent, they will think that the low-ability politician's equilibrium strategy assigned probability $s^B(\sigma_1)$ to playing a_1. The politician's expected payoff from playing a_0 is unaffected because $\mu(\sigma_0, a_0, 0) = \mu(\sigma_1, a_0, 0) = 0$;[13] it is still given by Eq. (5). His payoff from a_1, however, becomes

$$\phi\nu_0 + (1-\phi)[\nu_0\mu(\sigma_1, a_1, 1) + (1-\nu_0)\mu(\sigma_0, a_1, 0)]$$
$$= \phi\nu_0 + (1-\phi)\frac{\nu_0\lambda}{\lambda + (1-\lambda)s^B(\sigma_1)}. \tag{8}$$

Similarly, consider the politician's decision problem when $\sigma = \sigma_1$. If he chooses policy a_0 and succeeds ($y=1$) or chooses a_1 and fails ($y=0$), voters' recalled signal will be $\hat{\sigma} = \sigma_0$. When forming their posterior, they will think that the politician's strategy assigned probability $s^B(\sigma_0)$ to playing a_1. The politician's payoff from a_1 is unaffected (because $\mu(\sigma_1, a_1, 0) = \mu(\sigma_0, a_1, 0) = 0$) and thus given by Eq. (6). His payoff from a_0, however, becomes

$$\phi[p(1-\nu_1) + (1-p)\kappa] + (1-\phi)[p(\nu_1\mu(\sigma_1, a_0, 0) + (1-\nu_1)\mu(\sigma_0, a_0, 1))$$
$$+ (1-p)\mu(\sigma_1, a_0, \kappa)] = \phi[p(1-\nu_1) + (1-p)\kappa]$$
$$+ (1-\phi)\left[\frac{p(1-\nu_1)\lambda}{\lambda + (1-\lambda)(1-s^B(\sigma_0))} + \frac{(1-p)\lambda(1-\nu_1)}{\lambda(1-\nu_1) + (1-\lambda)(1-s^B(\sigma_1))}\right]. \tag{9}$$

Expressions (8) and (9) reveal the key difference between rational and hindsight-biased evaluation: with hindsight-biased voters, the politician's strategy in the σ_0 subgame depends on his strategy in the σ_1 subgame, and vice versa. [14] The following lemma characterizes the

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[13] This is partly due to the assumption of pessimistic out-of-equilibrium beliefs. The assumption is generally not crucial though. The only case where pessimistic beliefs matter is when $s(\sigma_1) = 1$ and $s(\sigma_0) < 1$. Without the pessimistic belief assumption pinning down out-of-equilibrium beliefs, $\mu(\sigma_1, a_0, 0)$ could be strictly positive in that case. Hindsight bias would then *increase* the incumbent's reputation in the case of failure. Such a result is counter-intuitive: we would expect hindsight bias to lead voters to second-guess politicians' decisions and give more blame than is warranted, rather than the opposite. Any intuitive setup for hindsight bias requires out-of-equilibrium beliefs to be pessimistic.

[14] This is true except when $p=0$. In that case, the term that depends on $s(\sigma_0)$ in Eq. (9) disappears, and $s^B(\sigma_1) = s^R(\sigma_0)$.

resulting equilibrium with hindsight-biased evaluation, for which $s^B(\sigma)$ denotes the low type's equilibrium strategy.

Lemma 3. Suppose voters are hindsight biased. There exists a unique equilibrium characterized by four thresholds, $\underline{\nu}_0^B$, $\overline{\nu}_1^B$, $\underline{\nu}^R$, and $\overline{\nu}^R$, such that the high-ability politician chooses a_0 when $\omega = 0$ and a_1 when $\omega = 1$, while

  (i). in the σ_0 subgame, for $\nu_0 \leq \underline{\nu}_0^B$ the low-ability politician always chooses a_0 ($s^B(\sigma_0) = 0$), for $\nu_0 \geq \overline{\nu}^R$ he always chooses a_1 ($s^B(\sigma_0) = 1$), and for $\underline{\nu}_0^B < \nu_0 < \overline{\nu}^R$ he randomizes between a_0 and a_1: $s^B(\sigma_0) = S_0^B(\nu_0, \nu_1)$ where $S_0^B(\nu_0, \nu_1) \in (0,1)$ increases with ν_0 and decreases with ν_1;

  (ii). in the σ_1 subgame, for $\nu_1 \leq \underline{\nu}^R$ the low-ability politician always chooses a_0 ($s^B(\sigma_1) = 0$), for $\nu_1 \geq \overline{\nu}_1^B$ he always chooses a_1 ($s^B(\sigma_1) = 1$), and for $\underline{\nu}^R < \nu_1 < \overline{\nu}_1^B$ he randomizes between a_0 and a_1: $s^B(\sigma_1) = S_1^B(\nu_0, \nu_1)$ where $S_1^B(\nu_0, \nu_1) \in (0,1)$ decreases with ν_0 and increases with ν_1.

Even under hindsight-biased evaluation, the high type follows his information and chooses the welfare-maximizing policy.[15] Moreover, the low type's equilibrium strategy again depends on ν. In contrast to the equilibrium under rational evaluation, there are now two sets of thresholds, one for each subgame. In addition, both the thresholds and the mixing probabilities in each subgame now depend on both ν_0 and ν_1, as shown in Fig. 3. The figure plots ν_0 on the horizontal axis and ν_1 on the vertical axis, and shows the low type's equilibrium strategy $s^B(\sigma)$ in each of the two subgames as a function of ν_0 and ν_1. The shaded triangle below the 45-degree line represents values such that $\nu_0 > \nu_1$, which are ruled out by Assumption 1. Fixing ν_1, it remains true that for low levels of ν_0, we have a pure-strategy equilibrium in which the low type plays a_0, for high levels of ν_0, we have a pure-strategy equilibrium in which the low type plays a_1, and for intermediate values, we have a mixed-strategy equilibrium. However, while in the case of rational evaluation, the equilibrium in the σ_0 subgame, say, is pinned down by ν_0 only, in the case of hindsight-biased evaluation, it also depends on ν_1. Fixing ν_0, the low type's equilibrium probability of playing a_1 decreases with ν_1.

An important observation that is apparent from Fig. 3 is that, whatever ν_0 and ν_1, we always have $s^B(\sigma_0) \leq s^B(\sigma_1)$. As we show in the proof of Lemma 3, this result follows from a consistency argument. Under Assumption 1, $\omega = 1$ is more likely after σ_1 than after σ_0, and therefore the expected welfare from a_1 is higher after σ_1 as well. Even with hindsight-biased voters, in equilibrium it cannot be the case that a_1 is played more often after σ_0 than after σ_1. Having characterized the equilibrium under hindsight biased evaluation, we now compare the low-ability politician's behavior to that under rational evaluation.

Proposition 2. Under hindsight-biased evaluation, the low-ability politician plays a_1 less often after σ_0 and more often after σ_1 than he does under rational evaluation: $s^B(\sigma_0) \leq s^R(\sigma_0)$ and $s^B(\sigma_1) \geq s^R(\sigma_1)$.

That is, hindsight bias increases the probability that the low-ability politician plays the action that—according to the signal—corresponds to the more likely state of the world: a_0 after σ_0, and a_1 after σ_1. The consistency argument mentioned above is crucial for this result. It implies that a politician who successfully enacts policy a_1 after signal σ_0 or policy a_0 after signal σ_1 will get less credit than he deserves. Voters consider the success as more predictable than it was, and therefore overestimate the probability that a low type would have chosen the policy. For example, when the original signal is σ_0 but the politician successfully implements policy a_1, voters wrongly believe that

Fig. 3. Equilibrium with hindsight-biased voters.

probability of the low type playing a_1 was $s^B(\sigma_1)$ instead of $s^B(\sigma_0)$. Because $s^B(\sigma_1) \geq s^B(\sigma_0)$, they exaggerate the likelihood that the observed event can be attributed to a low type, which reduces their esteem for the incumbent.

Anticipating the voters' bias, a low-ability politician knows that he has relatively little to gain from deviating to the action corresponding to the less likely state of the world, and does so less often. As the following proposition shows, this is often good for voters' first-period welfare, but can sometimes be bad.

Proposition 3. Let $\underline{\kappa} \equiv \dfrac{\nu_0(1 + p) - p}{1 - p}$ and $\overline{\kappa} \equiv \dfrac{\nu_1(1 + p) - p}{1 - p}$.

For $p > 0$, the effect of hindsight bias on first-period welfare is positive if $\underline{\kappa} \leq \kappa \leq \overline{\kappa}$, and ambiguous otherwise. For $p = 0$, the effect of hindsight bias on first-period welfare is positive if $\kappa \geq \underline{\kappa} = \nu_0$, and negative otherwise.

Note that $lim_{p\to1}\underline{\kappa} = -\infty$ and $lim_{p\to1}\overline{\kappa} = \infty$. Thus, as p increases, the range of values of κ for which the effect of hindsight bias is positive expands, eventually to encompass the entire $(0,1)$ interval. Based on this insight, we can rephrase Proposition 3 as follows. When p is large, hindsight bias is sure to enhance first-period welfare. When p is small, the effect of hindsight bias depends on κ. If κ is in an intermediate range, the effect of hindsight bias is positive; otherwise, its effect is ambiguous. When $p = 0$ and $\kappa < \nu_0$, hindsight bias is bad for first-period welfare; otherwise, its effect is positive.

According to Proposition 3, hindsight bias can enhance first-period welfare by improving the low-ability politician's discipline. Sometimes, however, it affects behavior in a way that is detrimental to welfare. To build intuition for which of the two is going to happen, it is again helpful to take a closer look at the two polar cases of $p = 0$ and $p = 1$. When $p = 1$, we have $\nu^* = 1/2$. Optimality requires that the low type plays a_1 after σ_1 and a_0 after σ_0. We know from Proposition 2 that hindsight bias nudges him in exactly this direction. Hence, for $p = 1$ the bias is welfare-enhancing.[16] By continuity, this also holds for some $p < 1$.

Hindsight bias eliminates some of the policy gambles that occur under rational evaluation. Recall that with rational voters, the reason that the low type sometimes chooses the inefficient policy is to

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[15] This means there are no adverse effects of hindsight bias on what Canes-Wrone et al. (2001) call "true leadership." That is, a scenario in which the high-ability politician no longer follows his private signal when voters are hindsight biased whereas he does when they are rational cannot happen. It can be shown that this result does not depend on Assumption 2 but holds more generally as long as the high-ability politician has perfect information.
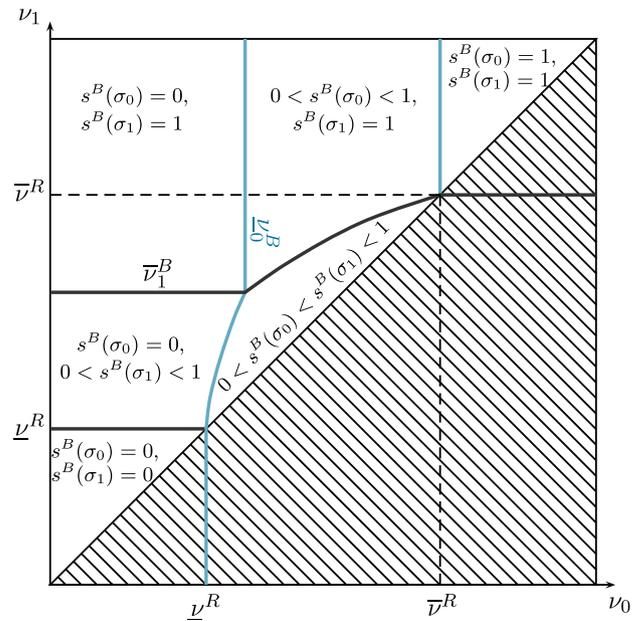
[16] In fact, for $p = 1$, hindsight bias leads the low type to choose the welfare-maximizing cutoff rule $\underline{\nu}_0^B = \overline{\nu}_1^B = \nu^*$.

impress voters: when a policy is unlikely to be played by a low type (as is the case when the signal indicates that the other policy leads to higher expected welfare), successfully enacting this policy boosts the politician's reputation. Rational voters are surprised that the policy was a success; they think that it must be attributed to a high type having superior private information and adjust their beliefs upward. Hindsight-biased voters are less easily impressed. They consider the policy outcome in retrospect more predictable than it actually was, and therefore think that choosing the right policy was "obvious" even for a low-ability politician. Put differently, hindsight-biased voters effectively forget their prior when they learn the state of the world, which takes away the possibility of going against public beliefs.

When $p = 0$, hindsight bias affects only the strategy in the σ_0 subgame. The σ_1 subgame is unaffected because voters do not learn anything after a_0, so hindsight bias only changes the recollection from σ_1 to σ_0 after a_1 and outcome $y = 0$. But $\mu(\sigma_0, a_1, 0) = \mu(\sigma_1, a_1, 0) = 0$, implying $s^B(\sigma_1) = s^R(\sigma_1)$ (the term in $s(\sigma_0)$ disappears from Eq. (9) when $p = 0$). From Proposition 2, we know that hindsight bias changes the strategy in the σ_0 subgame towards a greater probability of playing a_0. Whether this is good or bad for welfare depends on κ, which equals the expected payoff from a_0 when $p = 0$. The expected payoff from a_1 is ν_0. If $\kappa < \nu_0$, the expected payoff from a_0 is smaller than that from a_1, and by increasing the frequency at which a_0 is played, hindsight bias exacerbates inefficiencies. If $\kappa \geq \nu_0$, playing a_0 is efficient, so the bias has a positive effect.

The case $p = 0$, $\kappa < \nu_0$ illustrates a second effect of hindsight bias, namely, that it sometimes discourages *efficient* risk-taking. When $p = 0$, policy a_1 is risky while policy a_0 is safe, in the sense that the policy outcome after a_1 depends on the state of the world whereas after a_0 it does not. Because the low-ability politician knows that with hindsight-biased voters he will get less credit than he deserves for successfully enacting the risky policy a_1, he plays a_0 more often than with rational voters, independently of which policy is more efficient.

As we increase p slightly above zero, a countervailing effect appears that makes the effect of hindsight bias ambiguous. For low values of κ ($\kappa < \underline{\kappa}$), the bias continues to exacerbate excessive play of a_0 in the σ_0 subgame, but it now also affects the σ_1 subgame, where it reduces excessive play of a_0. For high values of κ ($\kappa > \overline{\kappa}$), playing a_0 is efficient, and so the effect of the bias is the opposite: in the σ_0 subgame, it reduces excessive play of a_1, while in the σ_1 subgame, it exacerbates it. Only for intermediate values of κ is hindsight bias unambiguously good: for $\underline{\kappa} < \kappa < \overline{\kappa}$ it nudges the low-ability politician toward the efficient choice in *both* subgames.

4. Discussion

The results in the previous section were derived under the assumption of naivety on the part of voters. In addition, the results focus on the effect of hindsight bias on discipline (and thus first-period welfare). In this section, we discuss the implications of relaxing the naivety assumption and the effect of hindsight bias on selection (and thus second-period welfare).

4.1. Awareness: naive versus sophisticated voters

According to Assumption 3, voters in our model are naive, i.e., ex post unaware of their bias: they are certain that they correctly remember the signal. The imperfect recollection process hinders conscious learning and implies a reduction in surprises. A natural extension is to depart from the assumption of naivety as, for example, in Bénabou and Tirole (2002), where individuals repress information but know about their tendency to repress. Suppose that evaluators are aware of their possibly distorted recollection of the ex ante signal at the voting stage, i.e., they are *sophisticated*. Voters will then use Bayes' rule to calculate the *reliability* of their recollection. The reliability of a voter's recollection is the probability that the recalled signal matches

the original signal. For example, suppose that a sophisticated voter observes policy a_1 and outcome $y = 1$, and recalls signal σ_1. He knows that, with some probability, the original signal was indeed σ_1, but with some probability it was actually σ_0 and has been replaced by σ_1 due to hindsight bias. Let $r_\omega \equiv Pr(\sigma = \sigma_\omega | \hat{\sigma} = \sigma_\omega, y = 1, \omega)$ denote the reliability of the voter's recollection given that the state of the world has been revealed to be ω and $y = 1$.[17] Two of the voter's posterior beliefs about the incumbent now have to be modified from Eq. (2). They are replaced by

$$\mu(\sigma_0, a_0, 1) = r_0 \frac{\lambda}{\lambda + (1-\lambda)(1-s(\sigma_0))} + (1-r_0) \frac{\lambda}{\lambda + (1-\lambda)(1-s(\sigma_1))},$$

$$\mu(\sigma_1, a_1, 1) = r_1 \frac{\lambda}{\lambda + (1-\lambda)s(\sigma_1)} + (1-r_1) \frac{\lambda}{\lambda + (1-\lambda)s(\sigma_0)};$$

all other beliefs are the same. Computing r_ω is straightforward, but it requires some additional notation and is therefore relegated to Appendix B. A sophisticated voter questions his own memory. When having recollection $\hat{\sigma} = \sigma_1$ after a successful policy a_1, for example, the voter realizes that the low-ability politician played a_1 with probability $s(\sigma_1)$ if the recollection is reliable (probability r_1) and with probability $s(\sigma_0)$ if the recollection is unreliable (probability $1 - r_1$).

How does the behavior of the low-ability politician change when he faces sophisticated rather than naive voters? Notice that the politician's reputation after enacting a successful policy is a convex combination of those with naive hindsight-biased and rational voters. Therefore, the effect of hindsight bias on his behavior is qualitatively the same as with naive voters. Sophistication lowers the impact of hindsight bias but does not eliminate it. The essence of the previous analysis remains intact.

4.2. Selection and welfare

The analysis in Section 3 shows how hindsight bias affects the behavior of the low-ability incumbent in the first period. To make a general statement about the impact of hindsight bias on welfare, however, we must also take into account the second period. Even though the absence of reelection concerns in the second period means that hindsight bias does not change behavior, the bias impacts second-period welfare through its effect on selection: voters are always weakly better off if a high-ability politician is in office.

The effect of hindsight bias on selection works through two channels. The first is that voters sometimes have erroneous posteriors, so that they don't always elect the politician that is truly more able (in expected terms); this is clearly bad for welfare. Hindsight bias blurs the difference between the two major elements voters initially set out to distinguish between in their evaluation, namely, skill and luck of the decision maker. The second is more indirect: since anticipation of voters' hindsight bias changes the low-ability politician's first-period behavior, the inferences that can be drawn from a given event are also modified. The welfare implications of this second effect are more complex. For example, hindsight bias increases the low type's equilibrium probability of playing a_0 after receiving σ_0. This decreases the posterior probability that the incumbent is a high type after observing a_0 and either $y = 1$ or $y = \kappa$, while it increases the posterior after observing a_1 and $y = 1$. The net effect on selection can go either way and depends on various parameters such as the prior beliefs about the state of the world.

Note that, even if hindsight bias is bad for selection, it can still be good for overall welfare. In particular, suppose $\underline{\kappa} < \kappa < \overline{\kappa}$, so that by Proposition 3, hindsight bias has a disciplining effect on the low-ability politician. Then, there exists a discount factor such that hindsight bias is overall welfare-enhancing. Because voters discount

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[17] Reliability can be defined analogously for $y = 0$. However, when $y = 0$, posterior beliefs are zero regardless of the original signal, so reliability does not matter.

the future, discipline is more important than selection for a sufficiently low discount factor.

5. Conclusion

We have presented a political-agency model in which voters exhibit a cognitive deficiency known as hindsight bias: after the uncertainty about an event is resolved, they consider the realized outcome as being more foreseeable than it actually was. In the model, voters have to evaluate the incumbent in order to decide whether to reelect him or replace him with a challenger. Politicians are assumed to differ in ability, where ability is defined as the quality of their information about the welfare-maximizing policy. High-ability politicians are better informed than low-ability politicians and voters. In this setup, low-ability politicians have incentives to disregard public information on what the optimal policy is in order to appear to have superior private information. Thus, they sometimes gamble on inefficient policies.

We have shown that hindsight bias on the part of voters can act as a discipline device. This is because hindsight-biased voters are less easily impressed by a successful gamble – they think it was the obvious choice from the outset, even if the available information had suggested otherwise. Therefore, they give an incumbent who succeeds with a policy gamble in spite of public pessimism less credit than rational voters who perfectly recall their prior. Anticipating this, low-ability politicians are typically less likely to deviate from the welfare-maximizing policy. In some circumstances, however, the bias can also discourage efficient risk taking, which is detrimental to voters' welfare.

We now return to the Gulf War anecdote brought up in the introduction and discuss why it supports our claim that hindsight bias can affect elections. A case can be made that hindsight bias contributed to George H.W. Bush's defeat in the 1992 presidential election. With Bayesian voters, Bush's success in Iraq would have shown up positively in his foreign-policy approval rate. But to hindsight-biased voters, the use of military force seemed an obvious choice. While in the immediate post-war euphoria, approval for the President's foreign policy did go up, by April of 1992 it was back to (or even slightly below) its pre-war level.[18] Voters, in retrospect, did not give Bush much credit for the successful operation in the Gulf. Their hindsight bias must have been detrimental to his chances in the November 1992 election, which he lost to Bill Clinton.[19]

Our analysis may be applicable to problems beyond political economy. For example, much like elections, promotion decisions in organizations do not follow rules set forth in an explicit ex ante contract. Consider a human resources department that has to decide whether to promote an employee from inside the firm, whose actions and performance have been observed, or to hire an outsider for the job. In a firm, there is typically a certain amount of public information concerning the way an employee is supposed to handle his task (in terms of the model, what the right action is), but employees may also have superior information on their specific assignment. Our model would predict that, if anticipated, hindsight bias on the part of the human resources department may prevent low-ability employees from deviating to suboptimal actions in order to appear smart, but would not necessarily help in choosing the right candidate.

We close by noting that, with the benefit of hindsight, all of our results are, of course, obvious.

[18] See Mueller (1994, Table 3); approval rates for Bush's handling of the situation in the Middle East follow the same pattern. The gradual decline in the approval rate is consistent with experimental evidence according to which hindsight bias increases over time (Bryant and Brockway, 1997).

[19] Admittedly, there is some question as to whether the war in Iraq was an important factor in the voters' election decision. Although as late as September 1992, almost 70% of likely voters indeed said that the war was important (Mueller, 1994, Table 282), political scientists have come to view the election as largely decided by issues other than foreign policy. Notice, however, that this is not inconsistent with hindsight bias having influenced the election. In fact, if voters thought that the decision to use military force was a "no-brainer," its favorable outcome should not have played much of a role in their updating of the President's perceived competence, compared to other, seemingly more informative issues, such as his handling of the economy.

Appendix A. Proofs

Proof of Lemma 1. We have to consider two types of equilibria: pooling equilibria, and equilibria in which the high type plays $a_{1-\omega}$.

Claim 1. *There cannot be a pooling equilibrium.* Consider the following sets of strategies and beliefs: (i) all types pool on a_0, and voters believe that any politician who plays a_1 is of type θ_L, i.e., $\mu(\sigma, a_1, y) = 0 \ \forall \sigma, y$; (ii) all types pool on a_1, and voters believe that any politician who plays a_0 is of type θ_L, i.e., $\mu(\sigma, a_0, y) = 0 \ \forall \sigma, y$. Regardless of σ, the type with the strongest incentive to deviate from pooling on a_0 is $(\theta_H, 1)$. Playing a_0 procures him $\phi(1-p)\kappa + (1-\phi)\lambda$, while playing a_1 would procure him ϕ. Assumption 2 implies that he prefers a_1. The type with the strongest incentive to deviate from pooling on a_1 is $(\theta_H, 0)$. Playing a_1 procures him $(1-\phi)\lambda$, while playing a_0 would procure him $\phi[(1-p)\kappa + p]$. Again, Assumption 2 implies that he prefers a_0.

Claim 2. *There cannot be an equilibrium in which the high type plays $a_{1-\omega}$.*
We show first that, whatever the voters' beliefs, $(\theta_H, 1)$ has a stronger incentive to play a_1 than θ_L, and θ_L has a stronger incentive to play a_1 than $(\theta_H, 0)$. The difference in expected payoffs between playing a_1 and a_0 for type θ_L is given by

$$\phi\nu + (1-\phi)[\nu\mu(\sigma, a_1, 1) + (1-\nu)\mu(\sigma, a_1, 0)] - \phi[p(1-\nu) + (1-p)\kappa] - (1-\phi)[p(\nu\mu(\sigma, a_0, 0) + (1-\nu)\mu(\sigma, a_0, 1)) + (1-p)\mu(\sigma, a_0, \kappa))]. \tag{10}$$

The difference in expected payoffs between a_1 and a_0 for type $(\theta_H, 0)$ is given by

$$(1-\phi)\mu(\sigma, a_1, 0) - \phi[p + (1-p)\kappa] - (1-\phi)[p\mu(\sigma, a_0, 1) + (1-p)\mu(\sigma, a_0, \kappa)], \tag{11}$$

and for type $(\theta_H, 1)$ by

$$\phi + (1-\phi)\mu(\sigma, a_1, 1) - \phi(1-p)\kappa - (1-\phi)[p\mu(\sigma, a_0, 0) + (1-p)\mu(\sigma, a_0, \kappa)]. \tag{12}$$

Substracting Eq. (11) from Eq. (10), we see that θ_L's incentive to play a_1 is greater than $(\theta_H, 0)$'s if and only if $\phi(1+p) + (1-\phi) [\mu(\sigma, a_1, 1) - \mu(\sigma, a_1, 0) + p(\mu(\sigma, a_0, 1) - \mu(\sigma, a_0, 0))] > 0$. Subtracting Eq. (10) from Eq. (12), we see that $(\theta_H, 1)$'s incentive to play a_1 is greater than θ_L's under exactly the same condition. Since the term in square brackets cannot be smaller than $-(1+p)$ for any of the voters' beliefs, under Assumption 2 this condition is always satisfied. We can thus conclude that: if θ_L is indifferent, θ_H strictly prefers a_ω; if θ_L strictly prefers a_0, so does $(\theta_H, 0)$; and if θ_L strictly prefers a_1, so does type $(\theta_H, 1)$. Hence, there cannot be an equilibrium where both $(\theta_H, 0)$ and $(\theta_H, 1)$ play $a_{1-\omega}$. $\square$

Proof of Lemma 2. By Lemma 1, any equilibrium, if it exists, must have the high-ability politician choosing the welfare-maximizing policy. Taking the behavior of the high type as given, we derive conditions for pure and mixed strategy equilibria on the part of the low-ability politician. We then show that, given the low type's behavior and voters' beliefs, the high-ability politician indeed finds it optimal to follow the claimed equilibrium strategy.

Define $\Delta(\nu) \equiv \frac{\phi}{1-\phi}[p(1-\nu) + (1-p)\kappa - \nu]$. A pure-strategy equilibrium with $s(\sigma) = 0$ requires that the low type prefers a_0, given by Eq. (5), even though he could fool voters about his type by playing a_1, given by Eq. (6), when evaluating both expressions at $s(\sigma) = 0$. That is,

$$\phi[(1-p)\kappa + p(1-\nu)] + (1-\phi)\left[(1-p)\frac{\lambda(1-\nu)}{1-\lambda\nu} + p(1-\nu)\lambda\right] \geq \nu[\phi + (1-\phi)] \tag{13}$$

$$\Leftrightarrow \Delta(\nu) \geq \nu - \left[p\lambda(1-\nu) + \frac{(1-p)\lambda(1-\nu)}{1-\lambda\nu} \right]. \tag{14}$$

The left-hand side (LHS) is decreasing and the right-hand side (RHS) increasing in ν. Moreover, the LHS is positive and the RHS negative when evaluated at $\nu = 0$. The LHS is negative and the RHS positive at $\nu = 1$. Hence there exists a unique cutoff $\underline{\nu}^R \in (0,1)$ below which $s^R(\sigma) = 0$ is an equilibrium. The cutoff value $\overline{\nu}^R$ is defined by expression (14) holding with equality. A necessary and sufficient condition for a pure-strategy equilibrium where $s(\sigma) = 1$ is that type θ_L prefers a_1 even though voters believe that he will always play a_1. This condition is obtained by evaluating Eqs. (5) and (6) at $s(\sigma) = 1$. That is,

$$\phi[(1-p)\kappa + p(1-\nu)] + (1-\phi)[(1-p) + p(1-\nu)] \leq \nu[\phi + (1-\phi)\lambda] \tag{15}$$

$$\Leftrightarrow \Delta(\nu) \leq \nu(\lambda + p) - 1. \tag{16}$$

The LHS is decreasing and the RHS is increasing in ν. Moreover, the LHS is positive and the RHS negative when evaluated at $\nu = 0$. Hence there exists a unique cutoff $\overline{\nu}^R > 0$ above which $s^R(\sigma) = 1$ is an equilibrium. The cutoff value $\overline{\nu}^R$ is defined by expression (16) holding with equality.

Uniqueness follows from Lemma 1 and the fact that $\underline{\nu}^R < \overline{\nu}^R$. To see that this inequality must hold, notice that the LHS of expressions (14) and (16) coincides and is monotone decreasing in ν. At the same time, the RHS of Eq. (14) exceeds that of Eq. (16),

$$\nu(1 + p\lambda) - \left[p\lambda + (1-p)\frac{\lambda(1-\nu)}{1-\lambda\nu} \right] > \nu(\lambda + p) - 1,$$

because the term in square brackets is smaller than 1 and $1 + p\lambda > \lambda + p \Leftrightarrow \lambda + (1-\lambda)p < 1$.

For values $\underline{\nu}^R < \nu < \overline{\nu}^R$ the equilibrium is in mixed strategies, where $s^R(\sigma) = S^R(\nu) \in (0,1)$. Type θ_L's equilibrium probability of playing a_1, $S^R(\nu)$, is determined by equating Eqs. (3) and (4), or

$$\Delta(\nu) = \frac{\nu\lambda}{\lambda + (1-\lambda)S^R(\nu)} - \frac{(1-p)\lambda(1-\nu)}{\lambda(1-\nu) + (1-\lambda)(1-S^R(\nu))}$$
$$- \frac{p\lambda(1-\nu)}{\lambda + (1-\lambda)(1-S^R(\nu))}. \tag{17}$$

Next, we prove the claimed monotonicity property of $S^R(\nu)$ by applying the implicit function theorem. Define $f \equiv LHS - RHS$ of Eq. (17). We then have $\frac{dS^R(\nu)}{d\nu} = -\frac{\partial f / \partial \nu}{\partial f / \partial S^R}$. It is straightforward to see that $\partial f / \partial S^R > 0$ and $\partial f / \partial \nu < 0$. Hence, $dS^R/d\nu > 0$.

We finally show that, given the voters' beliefs, it is indeed optimal for the high-ability politician to choose the policy corresponding to ω. Consider first type $(\theta_H, 0)$. He prefers a_0 since

$$\Delta(0) + \left[\frac{(1-p)\lambda(1-\nu)}{\lambda(1-\nu) + (1-\lambda)(1-s^R(\sigma))} + \frac{p\lambda}{\lambda + (1-\lambda)(1-s^R(\sigma))} \right] > 0 \tag{18}$$

for any $s^R(\sigma)$. Now consider type $(\theta_H, 1)$. The condition for him to prefer a_1 to a_0 is

$$\Delta(1) \leq \frac{\lambda}{\lambda + (1-\lambda)s^R(\sigma)} - \frac{(1-p)\lambda(1-\nu)}{\lambda(1-\nu) + (1-\lambda)(1-s^R(\sigma))}. \tag{19}$$

Clearly, $\Delta(1) < 0$. There are three different cases. If $\nu < \underline{\nu}^R$ so that $s^R(\sigma) = 0$, type $(\theta_H, 1)$ has a strict preference for a_1 because $\Delta(1) < \frac{1 - \lambda + p\lambda(1-\nu)}{1-\lambda\nu}$. If $\nu > \overline{\nu}^R$ so that $s^R(\sigma) = 1$, he prefers a_1 because $\Delta(1) < \lambda + p - 1$ is a necessary condition for the low type to play a pure strategy with $s(\sigma) = 1$; see Eq. (16). If $\underline{\nu}^R < \nu < \overline{\nu}^R$ we are in a mixed strategy equilibrium where $s^R(\sigma) = S^R(\nu)$. Type $(\theta_H, 1)$ strictly prefers a_1 since $\Delta(1) < \frac{\lambda}{\lambda + (1-\lambda)S^R(\nu)} - \frac{(1-p)\lambda(1-\nu)}{\lambda(1-\nu) + (1-\lambda)(1-S^R(\nu))}$ follows from Eq. (17). $\qquad\square$

Proof of Proposition 1. Solving Eq. (16) for $\overline{\nu}^R$ yields $\overline{\nu}^R = \frac{\phi(p + (1-p)\kappa) + 1 - \phi}{\phi + (1-\phi)\lambda + p}$.

The numerator of this expression is larger than that of ν^* (it is a convex combination of $p + (1-p)\kappa$ and 1). The denominator is smaller than that of ν^* (because $\phi + (1-\phi)\lambda < 1$). Hence, $\nu^* < \overline{\nu}^R$. The RHS of Eq. (14) is monotone decreasing in ν and becomes zero at $\nu = \nu^*$. The LHS is monotone increasing in ν since its derivative with respect to ν is

$$1 + p\lambda + \frac{\lambda(1-\lambda)(1-p)}{(1-\lambda\nu)^2} > 0.$$

Thus, a necessary and sufficient condition for $\nu^* > \underline{\nu}^R$ is that the LHS of Eq. (14) is greater than zero when evaluated at $\nu = \nu^*$. That is,

$$\nu^* - \left[p(1-\nu^*)\lambda + \frac{(1-p)\lambda(1-\nu^*)}{1-\lambda\nu^*} \right] > 0 \tag{20}$$
$$\Leftrightarrow p\nu^*(1-\nu^*)\lambda^2 - [1 - \nu^*(1-\nu^*)]\lambda + \nu^* > 0.$$

The LHS of Eq. (20) is a quadratic function of λ whose minimum is attained at $\lambda = [1 - \nu^*(1-\nu^*)]/(2p\nu^*(1-\nu^*))$, which is greater or equal to 3/2. Therefore, the relevant condition is that $\lambda < \overline{\lambda}$ defined in Eq. (7). The expression under the square root is nonnegative since $(1 - \nu^*(1-\nu^*))^2 - 4p(\nu^*)^2(1-\nu^*) \geq (1 - \nu^*(1-\nu^*))^2 - 4(\nu^*)^2(1-\nu^*) = (\nu^*(1+\nu^*))^2$. Finally, evaluating Eq. (7) at $p = 1$ (which implies $\nu^* = 1/2$), we have $\overline{\lambda} = 1$. $\qquad\square$

Proof of Lemma 3. By Lemma 1, any equilibrium, if it exists, must have the high-ability politician choosing the welfare-maximizing policy. We thus look for an equilibrium in which this is the case and derive conditions for pure and mixed strategies on the part of the low-ability politician. We then show that, given the low type's behavior and voters' beliefs, the high-ability politician indeed finds it optimal to follow the equilibrium strategy.

Pure strategies. Consider first the σ_0 subgame. A necessary and sufficient condition for a pure-strategy equilibrium with $s^B(\sigma_0) = 0$ is that type θ_L's payoff from playing a_0, given by Eq. (5), is greater than

his payoff from a_1, given by Eq. (8), when evaluating both of these expressions at $s^B(\sigma_0) = 0$. That is,

$$\phi[p(1-\nu_0) + (1-p)\kappa] + (1-\phi)\left[p\lambda(1-\nu_0) + \frac{(1-p)\lambda(1-\nu_0)}{1-\lambda\nu_0}\right] \geq \phi\nu_0$$

$$+ (1-\phi)\frac{\nu_0\lambda}{\lambda + (1-\lambda)s^B(\sigma_1)}$$

$$\Leftrightarrow \Delta(\nu_0) \geq \frac{\nu_0\lambda}{\lambda + (1-\lambda)s^B(\sigma_1)} - p\lambda(1-\nu_0) - \frac{(1-p)\lambda(1-\nu_0)}{1-\lambda\nu_0}.$$

(21)

The LHS is decreasing in ν_0. The RHS is increasing in ν_0 if $s^B(\sigma_1)$ is decreasing in ν_0, which we will show to be the case below. Moreover, $\Delta(0) > 0$ and the RHS is negative when evaluated at $\nu_0 = 0$. Therefore, for a given ν_1, there exists a unique cutoff $\underline{\nu}_0^B > 0$ below which $s^B(\sigma_0) = 0$ is an equilibrium. (The cutoff may be greater than 1/2.)

The condition for a pure-strategy equilibrium with $s^B(\sigma_0) = 1$ is that type θ_L's payoff from a_0 is smaller than his payoff from a_1 when both are evaluated at $s^B(\sigma_0) = 1$, that is,

$$\phi[p(1-\nu_0) + (1-p)\kappa] + (1-\phi)[p(1-\nu_0) + 1-p] \leq \phi\nu_0 + (1-\phi)\frac{\nu_0\lambda}{\lambda + (1-\lambda)s^B(\sigma_1)}$$

$$\Leftrightarrow \Delta(\nu_0) \leq \nu_0\left[\frac{\lambda}{\lambda + (1-\lambda)s^B(\sigma_1)} + p\right] - 1.$$

(22)

Again, the LHS is decreasing and the RHS increasing in ν_0 if $s^B(\sigma_1)$ is decreasing in ν_0. Therefore, for a given ν_1, there exists a unique cutoff $\overline{\nu}_0^B$ above which $s^B(\sigma_0) = 1$ is an equilibrium. Note that for $s^B(\sigma_1) = 1$, Eq. (22) is the same as the corresponding expression in the rational case, Eq. (16), defining $\overline{\nu}^R$; note also that the RHS of Eq. (22) is decreasing in $s^B(\sigma_1)$, implying that $\overline{\nu}_0^B$ must equal $\overline{\nu}^R$ for $s^B(\sigma_1) = 1$ and would have to be strictly less than $\overline{\nu}^R$ for $s^B(\sigma_1) < 1$. Since, as we will show below, consistency requires $s^B(\sigma_0) \leq s^B(\sigma_1)$ for any (ν_0, ν_1) satisfying Assumption 1, we conclude that there cannot be a value of $\nu_0 < \overline{\nu}^R$ such that $s^B(\sigma_0) = 1$. This would require that for some $\nu_1 > \nu_0$, $s^B(\sigma_1) < 1$, a contradiction. Hence, $\overline{\nu}_0^B = \overline{\nu}^R$.

Now consider the σ_1 subgame. A necessary and sufficient condition for a pure-strategy equilibrium with $s^B(\sigma_1) = 0$ is that type θ_L's payoff from playing a_0, given by Eq. (9), is greater than his payoff from a_1, given by Eq. (6), when evaluating both of these expressions at $s^B(\sigma_1) = 0$. That is,

$$\phi[p(1-\nu_1) + (1-p)\kappa] + (1-\phi)\left[\frac{p\lambda(1-\nu_1)}{\lambda + (1-\lambda)(1-s^B(\sigma_0))} + \frac{(1-p)\lambda(1-\nu_1)}{1-\lambda\nu_1}\right]$$

$$\geq \phi\nu_1 + (1-\phi)\nu_1$$

$$\Leftrightarrow \Delta(\nu_1) \geq \nu_1 - \left[\frac{p\lambda(1-\nu_1)}{\lambda + (1-\lambda)(1-s^B(\sigma_0))} + \frac{(1-p)\lambda(1-\nu_1)}{1-\lambda\nu_1}\right].$$

(23)

The LHS is decreasing in ν_1. The RHS is increasing in ν_1 if $s^B(\sigma_0)$ is decreasing in ν_1, which we will show to be the case below. Therefore, for a given ν_0, there exists a unique cutoff $\underline{\nu}_1^B$ below which $s^B(\sigma_1) = 0$ is an equilibrium. (The cutoff may be smaller than 1/2.) Note that for $s^B(\sigma_0) = 0$, Eq. (23) is the same as the corresponding expression in the rational case, Eq. (14), defining $\underline{\nu}^R$; note also that the RHS of Eq. (23) is increasing in $s^B(\sigma_0)$, implying that $\underline{\nu}_1^B$ must equal $\underline{\nu}^R$ for $s^B(\sigma_0) = 0$ and would have to be strictly greater than $\underline{\nu}^R$ for $s^B(\sigma_1) < 1$. Since, as we will show below, consistency requires $s^B(\sigma_0) \leq s^B(\sigma_1)$ for any (ν_0, ν_1) satisfying Assumption 1, we conclude that there cannot be a value of $\nu_1 > \underline{\nu}^R$ such that $s^B(\sigma_1) = 0$. This would require that for some $\nu_0 < \nu_1$, $s^B(\sigma_0) > 0$, a contradiction. Hence, $\underline{\nu}_1^B = \underline{\nu}^R$.

The condition for a pure-strategy equilibrium with $s^B(\sigma_1) = 1$ is that type θ_L's payoff from a_0 is smaller than his payoff from a_1 when both are evaluated at $s^B(\sigma_1) = 1$, that is,

$$\phi[p(1-\nu_1) + (1-p)\kappa] + (1-\phi)\left[\frac{p\lambda(1-\nu_1)}{\lambda + (1-\lambda)(1-s^B(\sigma_0))} + 1-p\right] \leq \phi\nu_1 + (1-\phi)\nu_1\lambda$$

$$\Leftrightarrow \Delta(\nu_1) \leq \nu_1\lambda - \frac{p\lambda(1-\nu_1)}{\lambda + (1-\lambda)(1-s^B(\sigma_0))} - (1-p).$$

(24)

Again, the LHS is decreasing and the RHS increasing in ν_1 if $s^B(\sigma_0)$ is decreasing in ν_1. Therefore, for a given ν_0, there exists a unique cutoff $\overline{\nu}_1^B$ above which $s^B(\sigma_1) = 1$ is an equilibrium. (This cutoff may not be strictly less than 1.)

Uniqueness. Uniqueness of the equilibrium follows from the fact that (a) the RHS of Eq. (21) is greater than that of Eq. (22) for any ν_0 as

$$1 - p\nu_0 \geq \lambda(1-\nu_0)\left(p + \frac{1-p}{1-\lambda\nu_0}\right) \Leftrightarrow 1 - p\underbrace{\nu_0(1 + \lambda(1-\nu_0))}_{\leq 1} \geq 0,$$

implying $\underline{\nu}_0^B \leq \overline{\nu}_0^B$, and (b) the RHS of Eq. (21) is greater than that of Eq. (22) for any ν_1 as

$$\nu_1 - (1-p)\frac{\lambda(1-\nu_1)}{1-\lambda\nu_1} \geq \nu_1\lambda - (1-p) \Leftrightarrow \nu_1 \geq (1-p)\left(-\frac{1}{1-\lambda\nu_1}\right),$$

implying $\underline{\nu}_1^B \leq \overline{\nu}_1^B$.

Mixed strategies. In a mixed strategy equilibrium, we have $s^B(\sigma_0) = S_0^B(\nu_0, \nu_1)$ and $s^B(\sigma_1) = S_1^B(\nu_0, \nu_1)$, where S_0^B and S_1^B solve the following system, obtained by equating Eq. (5) with Eq. (8), and Eq. (9) with Eq. (6):

$$\Delta(\nu_0) = \frac{\nu_0\lambda}{\lambda + (1-\lambda)S_1^B} - \frac{p\lambda(1-\nu_0)}{\lambda + (1-\lambda)(1-S_0^B)} - \frac{(1-p)\lambda(1-\nu_0)}{\lambda(1-\nu_0) + (1-\lambda)(1-S_0^B)}$$

(25)

$$\Delta(\nu_1) = \frac{\nu_1\lambda}{\lambda + (1-\lambda)S_1^B} - \frac{p\lambda(1-\nu_1)}{\lambda + (1-\lambda)(1-S_0^B)} - \frac{(1-p)\lambda(1-\nu_1)}{\lambda(1-\nu_1) + (1-\lambda)(1-S_1^B)}.$$

(26)

To determine when there is mixing in at least one of the subgames, we need to define the thresholds $\underline{\nu}_0^B$ and $\overline{\nu}_1^B$ as a function of ν_1 and ν_0, respectively. Let $\hat{\nu}$ denote the value of ν_0 that solves Eq. (21) with equality for $s(\sigma_1) = 1$, i.e., $\Delta(\hat{\nu}) = \hat{\nu}\lambda - p\lambda(1-\hat{\nu}) - (1-p)\frac{\lambda(1-\hat{\nu})}{1-\lambda\hat{\nu}}$. Similarly, let $\tilde{\nu}$ denote the value of ν_1 that solves Eq. (24) with equality for $s(\sigma_0) = 0$, i.e., $\Delta(\tilde{\nu}) = \tilde{\nu}\lambda - p\lambda(1-\tilde{\nu}) - (1-p)$. Notice that, by definition of $\underline{\nu}_0^B$, $S_1^B(\underline{\nu}_0^B, \nu_1)$ is the solution to Eq. (26) fixing $S_0^B = 0$. Likewise, by definition of $\overline{\nu}_1^B$, $S_0^B(\nu_0, \overline{\nu}_1^B)$ is the solution to Eq. (25) fixing $S_1^B = 1$. Let $V_0(\nu_1)$ be defined as the solution to

$$\Delta(V_0) = \frac{V_0\lambda}{\lambda + (1-\lambda)S_1^B(\underline{\nu}_0^B, \nu_1)} - p\lambda(1-V_0) - \frac{(1-p)\lambda(1-V_0)}{1-\lambda V_0}, \quad (27)$$

and $V_1(\nu_0)$ as the solution to

$$\Delta(V_1) = V_1\lambda - \frac{p\lambda(1-V_1)}{\lambda + (1-\lambda)\left(1-S_0^B(\nu_0,\overline{\nu}_1^B)\right)} - (1-p). \tag{28}$$

Then, we have

$$\underline{\nu}_0^B = \begin{cases} \underline{\nu}^R & \text{for} \quad \nu_1 \le \underline{\nu}^R \\ V_0(\nu_1) & \text{for} \quad \underline{\nu}^R < \nu_1 < \tilde{\nu} \\ \hat{\nu} & \text{for} \quad \nu_1 \ge \tilde{\nu} \end{cases}$$

and

$$\overline{\nu}_1^B = \begin{cases} \tilde{\nu} & \text{for} \quad \nu_0 \le \hat{\nu} \\ V_1(\nu_0) & \text{for} \quad \hat{\nu} < \nu_0 < \overline{\nu}^R \\ \overline{\nu}^R & \text{for} \quad \nu_0 \ge \overline{\nu}^R. \end{cases}$$

Clearly, $\underline{\nu}^R \le V_0(\nu_1) \le \hat{\nu}$ for any ν_1, and $\tilde{\nu} \le V_1(\nu_0) \le \overline{\nu}^R$ for any ν_0.

Monotonicity properties of S^B. We now prove the claimed monotonicity properties of S_0^B and S_1^B. Let

$$f_0 \equiv \Delta(\nu_0) - \frac{\nu_0\lambda}{\lambda + (1-\lambda)S_1} + \frac{p\lambda(1-\nu_0)}{\lambda + (1-\lambda)(1-S_0)} + \frac{(1-p)\lambda(1-\nu_0)}{\lambda(1-\nu_0) + (1-\lambda)(1-S_0)},$$

$$f_1 \equiv \Delta(\nu_1) - \frac{\nu_1\lambda}{\lambda + (1-\lambda)S_1} + \frac{p\lambda(1-\nu_1)}{\lambda + (1-\lambda)(1-S_0)} + \frac{(1-p)\lambda(1-\nu_1)}{\lambda(1-\nu_1) + (1-\lambda)(1-S_1)}.$$

Suppose that (S_0^B, S_1^B) is an equilibrium for some (ν_0, ν_1), i.e., that it solves

$$f_0\left(S_0^B, S_1^B, \nu_0, \nu_1\right) = 0$$
$$f_1\left(S_0^B, S_1^B, \nu_0, \nu_1\right) = 0.$$

Totally differentiating, we have, in matrix notation,

$$\begin{pmatrix} \frac{\partial f_0}{\partial S_0} & \frac{\partial f_0}{\partial S_1} \\ \frac{\partial f_1}{\partial S_0} & \frac{\partial f_1}{\partial S_1} \end{pmatrix} \begin{pmatrix} dS_0 \\ dS_1 \end{pmatrix} = \begin{pmatrix} -\frac{\partial f_0}{\partial \nu_0}d\nu_0 - \frac{\partial f_0}{\partial \nu_1}d\nu_1 \\ -\frac{\partial f_1}{\partial \nu_0}d\nu_0 - \frac{\partial f_1}{\partial \nu_1}d\nu_1 \end{pmatrix}. \tag{29}$$

Clearly, $\partial f_\sigma/\partial S_{\sigma'} > 0$ for all σ and σ', while $\partial f_\sigma/\partial \nu_{\sigma'} < 0$ for all $\sigma = \sigma'$ and $\partial f_\sigma/\partial \nu_{\sigma'} = 0$ for all $\sigma \ne \sigma'$. Let

$$M \equiv \det \begin{pmatrix} \frac{\partial f_0}{\partial S_0} & \frac{\partial f_0}{\partial S_1} \\ \frac{\partial f_1}{\partial S_0} & \frac{\partial f_1}{\partial S_1} \end{pmatrix} = \frac{\partial f_0}{\partial S_0}\frac{\partial f_1}{\partial S_1} - \frac{\partial f_0}{\partial S_1}\frac{\partial f_1}{\partial S_0}.$$

Computation yields that $M > 0$ if and only if

$$\left(\underbrace{\frac{p(1-\nu_0)}{(\lambda + (1-\lambda)(1-S_0))^2}}_{\equiv A} + \underbrace{\frac{(1-p)(1-\nu_0)}{(\lambda(1-\nu_0) + (1-\lambda)(1-S_0))^2}}_{\equiv B}\right)$$
$$\cdot \left(\underbrace{\frac{\nu_1}{(\lambda + (1-\lambda)S_1)^2}}_{\equiv C} + \underbrace{\frac{(1-p)(1-\nu_1)}{(\lambda(1-\nu_1) + (1-\lambda)(1-S_1))^2}}_{\equiv D}\right) \tag{30}$$
$$> \underbrace{\frac{\nu_0}{(\lambda + (1-\lambda)S_1)^2}}_{\equiv E} \cdot \underbrace{\frac{p(1-\nu_1)}{(\lambda + (1-\lambda)(1-S_0))^2}}_{\equiv F}.$$

Under Assumption 1, $A \ge F$ and $C > E$, while $B, D \ge 0$. Therefore, $M > 0$, and we can invert Eq. (29) to obtain

$$\begin{pmatrix} dS_0 \\ dS_1 \end{pmatrix} = \frac{1}{M} \begin{pmatrix} \frac{\partial f_1}{\partial S_1} & -\frac{\partial f_0}{\partial S_1} \\ -\frac{\partial f_1}{\partial S_0} & \frac{\partial f_0}{\partial S_0} \end{pmatrix} \begin{pmatrix} -\frac{\partial f_0}{\partial \nu_0}d\nu_0 \\ -\frac{\partial f_1}{\partial \nu_1}d\nu_1 \end{pmatrix}. \tag{31}$$

This implies

$$\frac{dS_0}{d\nu_0} = \frac{1}{M}\frac{\partial f_1}{\partial S_1}\left(-\frac{\partial f_0}{\partial \nu_0}\right) > 0 \tag{32}$$

$$\frac{dS_0}{d\nu_1} = \frac{1}{M}\left(-\frac{\partial f_0}{\partial S_1}\right)\left(-\frac{\partial f_1}{\partial \nu_1}\right) < 0 \tag{33}$$

$$\frac{dS_1}{d\nu_0} = \frac{1}{M}\left(-\frac{\partial f_1}{\partial S_0}\right)\left(-\frac{\partial f_0}{\partial \nu_0}\right) < 0 \tag{34}$$

$$\frac{dS_1}{d\nu_1} = \frac{1}{M}\frac{\partial f_0}{\partial S_0}\left(-\frac{\partial f_1}{\partial \nu_1}\right) > 0, \tag{35}$$

establishing the claimed monotonicity properties.

Consistency. Next, we turn to the consistency condition according to which $s^B(\sigma_0) \le s^B(\sigma_1)$ for any pair (ν_0, ν_1). Suppose first $\nu_0 \le \underline{\nu}_0^B$. Then, $s^B(\sigma_0) = 0$. Similarly, suppose $\nu_1 \ge \overline{\nu}_1^B$. Then, $s^B(\sigma_1) = 1$. In both cases consistency is trivially satisfied. Next suppose $\nu_0 > \underline{\nu}_0^B$ and $\nu_1 < \overline{\nu}_1^B$, so that Eq. (25) holds with $\le$ and Eq. (26) holds with $\ge$. Subtracting Eq. (26) from Eq. (25), we then have, after simplifying,

$$(\nu_1 - \nu_0)\left[\frac{p\lambda}{\lambda + (1-\lambda)(1-s^B(\sigma_0))} + \frac{\lambda}{\lambda + (1-\lambda)s^B(\sigma_1)} + \frac{\phi}{1-\phi}(1+p)\right]$$
$$\le \frac{(1-p)\lambda(1-\nu_1)}{\lambda(1-\nu_1) + (1-\lambda)(1-s^B(\sigma_1))} - \frac{(1-p)\lambda(1-\nu_0)}{\lambda(1-\nu_0) + (1-\lambda)(1-s^B(\sigma_0))}. \tag{36}$$

The LHS is strictly positive because $\nu_1 > \nu_0$ by Assumption 1. Thus, a necessary condition for the inequality to hold is that the RHS be strictly positive as well. Suppose that contrary to the consistency requirement we have $s^B(\sigma_0) > s^B(\sigma_1)$. This implies $(1-\nu_1)(1-\lambda)$

$(1-s^B(\sigma_0))<(1-\nu_0)(1-\lambda)(1-s^B(\sigma_1))$. Adding $\lambda(1-\nu_0)(1-\nu_1)$ to both sides of the inequality, and rearranging, we obtain

$$\frac{1-\nu_1}{\lambda(1-\nu_1)+(1-\lambda)(1-s^B(\sigma_1))} < \frac{1-\nu_0}{\lambda(1-\nu_0)+(1-\lambda)(1-s^B(\sigma_0))},$$

a contradiction with the RHS of Eq. (36) being positive.

The high-ability politician. Finally, we examine the high-ability politician's incentive to adhere to the prescribed equilibrium strategy. Consider type $(\theta_H,0)$. He prefers a_0 since

$$\phi[p+(1-p)\kappa]+(1-\phi)\left[\frac{p\lambda}{\lambda+(1-\lambda)(1-s^B(\sigma_0))}+\frac{(1-p)\lambda(1-\nu)}{\lambda(1-\nu)+(1-\lambda)(1-s^B(\sigma_0))}\right]>0$$

for any $s^B(\sigma_0)$. Next, consider type $(\theta_H,1)$. The condition for him to prefer a_1 to a_0 is

$$\phi(1-p)\kappa+(1-\phi)\frac{(1-p)\lambda(1-\nu)}{\lambda(1-\nu)+(1-\lambda)(1-s^B(\sigma_0))}\leq\phi$$
$$+(1-\phi)\frac{\lambda}{\lambda+(1-\lambda)s^B(\sigma_1)}$$
$$\Leftrightarrow\Delta(1)\leq\frac{\lambda}{\lambda+(1-\lambda)s^B(\sigma_1)}-\frac{(1-p)\lambda(1-\nu)}{\lambda(1-\nu)+(1-\lambda)(1-s^B(\sigma_0))}.$$
$$(37)$$

There are several different cases. Suppose first that $s^B(\sigma_0)=0$. The RHS of Eq. (37) becomes

$$\underbrace{\frac{\lambda}{\lambda+(1-\lambda)s^B(\sigma_1)}}_{\geq\lambda}-\underbrace{\frac{(1-p)\lambda(1-\nu)}{1-\lambda\nu}}_{\leq\lambda}\geq0.$$

Now suppose that $s^B(\sigma_1)=0$. The RHS of Eq. (37) becomes $1-\frac{(1-p)\lambda(1-\nu)}{\lambda(1-\nu)+(1-\lambda)(1-s^B(\sigma_0))}\geq0$. Thus, if either $s^B(\sigma_0)$ or $s^B(\sigma_1)$ equals zero, Eq. (37) is satisfied because $\Delta(1)<0$. Suppose instead that neither equals zero, and consider the σ_0 subgame. If $0<s^B(\sigma_0)<1$, Eq. (25) must hold; if instead $s^B(\sigma_0)=1$, Eq. (22) must hold. Each implies Eq. (37). Now consider the σ_1 subgame. If $0<s^B(\sigma_1)<1$, Eq. (26) must hold; if instead $s^B(\sigma_1)=1$, Eq. (24) must hold. Again, each implies Eq. (37). $\square$

Proof of Proposition 2. We know from the proof of Lemma 3 that $\underline{\nu}^R=\underline{\nu}_1^B\leq\underline{\nu}_0^B$ and $\overline{\nu}_1^B\leq\overline{\nu}_0^B=\overline{\nu}^R$. Thus, if $\nu\leq\underline{\nu}^R$, then $s^B(\sigma)=s^R(\sigma)=0$, and if $\nu\geq\overline{\nu}^R$, then $s^B(\sigma)=s^R(\sigma)=1$. For the rest of the proof, we only have to consider values of ν in the interval $\left(\underline{\nu}^R,\overline{\nu}^R\right)$.

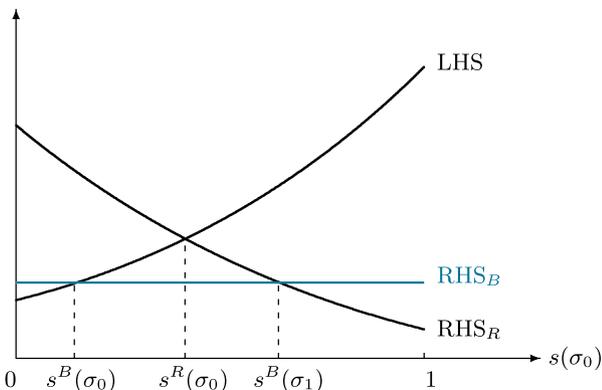**Fig. 4.** Determination of the low-type's equilibrium strategy in the σ_0 subgame.

Claim 1. *Suppose* $\underline{\nu}^R<\nu_0<\overline{\nu}^R$. *Then,* $s^B(\sigma_0)\leq s^R(\sigma_0)$ *if and only if* $s^B(\sigma_1)\geq s^R(\sigma_0)$.

When $\underline{\nu}^R<\nu_0<\overline{\nu}^R$, the rational-evaluation equilibrium in the σ_0-subgame is in mixed strategies, i.e., $s^R(\sigma_0)=S^R(\nu_0)$ where $S^R\in(0,1)$ is determined by Eq. (17); see Lemma 2. Writing out Eq. (17) for $\sigma=\sigma_0$ and rearranging, we have

$$\frac{p\lambda(1-\nu_0)}{\lambda+(1-\lambda)(1-s(\sigma_0))}+\frac{(1-p)\lambda(1-\nu_0)}{\lambda(1-\nu_0)+(1-\lambda)(1-s(\sigma_0))}+\Delta(\nu_0)=\frac{\nu_0\lambda}{\lambda+(1-\lambda)s(\sigma_0)}.$$
$$(38)$$

The equilibrium under hindsight-biased evaluation is determined by the following expression:

$$\frac{p\lambda(1-\nu_0)}{\lambda+(1-\lambda)(1-s(\sigma_0))}+\frac{(1-p)\lambda(1-\nu_0)}{\lambda(1-\nu_0)+(1-\lambda)(1-s(\sigma_0))}+\Delta(\nu_0)\lesseqgtr\frac{\nu_0\lambda}{\lambda+(1-\lambda)s^B(\sigma_1)}.$$
$$(39)$$

Fig. 4, which plots the left- and RHSs of expressions (38) and (39) against $s(\sigma_0)$, illustrates the logic of the argument that follows. The LHS of Eqs. (38) and (39) coincides and is monotone increasing in $s(\sigma_0)$. Under rational evaluation, the RHS, denoted RHS$_R$ in the figure, is monotone decreasing in $s(\sigma_0)$. The equilibrium strategy $s_R^*(\sigma_0)$ lies at the intersection of the LHS with the RHS$_R$ curve. Under hindsight-biased evaluation, the RHS is constant with respect to $s(\sigma_0)$; its value is obtained by evaluating RHS$_R$ at $s^B(\sigma_1)$, yielding the horizontal line denoted RHS$_B$ in the figure. The equilibrium strategy $s^B(\sigma_0)$ lies at the intersection of the LHS with the RHS$_B$ curve if such an intersection exists. If LHS$\geq$RHS$_B$ for all $s(\sigma_0)$, then $s^B(\sigma_0)=0$. If LHS$\leq$RHS$_B$ for all $s(\sigma_0)$, then $s^B(\sigma_0)=1$. As the figure clearly shows, the monotonicity properties of the LHS and RHS expressions imply that the claimed relationship must hold.

Claim 2. *Suppose* $\underline{\nu}^R<\nu_1<\overline{\nu}^R$. *Then,* $s^B(\sigma_1)\geq s^R(\sigma_1)$ *if and only if* $s^B(\sigma_0)\leq s^R(\sigma_1)$.

When $\underline{\nu}^R<\nu_1<\overline{\nu}^R$, the rational-evaluation equilibrium in the σ_1-subgame is in mixed strategies and again determined by Eq. (17). After rearranging, Eq. (17) yields, for $\sigma=\sigma_1$,

$$\frac{p\lambda(1-\nu_1)}{\lambda+(1-\lambda)(1-s(\sigma_1))}=\frac{\nu_1\lambda}{\lambda+(1-\lambda)s(\sigma_1)}-\frac{(1-p)\lambda(1-\nu_1)}{\lambda(1-\nu_1)+(1-\lambda)(1-s(\sigma_1))}-\Delta(\nu_1).$$
$$(40)$$

The equilibrium under hindsight-biased evaluation is determined by the following expression:

$$\frac{p\lambda(1-\nu_1)}{\lambda+(1-\lambda)(1-s^B(\sigma_0))}\lesseqgtr\frac{\nu_1\lambda}{\lambda+(1-\lambda)s(\sigma_1)}-\frac{(1-p)\lambda(1-\nu_1)}{\lambda(1-\nu_1)+(1-\lambda)(1-s(\sigma_1))}-\Delta(\nu_1).$$
$$(41)$$

The RHS of Eqs. (40) and (41) coincides and is monotone decreasing in $s(\sigma_1)$. The LHS of Eq. (40) is monotone increasing in $s(\sigma_1)$. The LHS of Eq. (41) is constant with respect to $s(\sigma_1)$; its value is obtained by evaluating the RHS of Eq. (40) at $s^B(\sigma_0)$. An analogous argument to the one used in the proof of Claim 1 establishes Claim 2. Combining Claims 1 and 2 with the consistency result from the proof of Lemma 3, we must have $s^B(\sigma_0)\leq s^R(\sigma)\leq s^B(\sigma_1)$, $\sigma=\sigma_0,\sigma_1$. Given that $s^R(\sigma_0)\leq s^R(\sigma_1)$ by Lemma 2 and Assumption 1, the only possible ordering of equilibrium strategies thus is $s^B(\sigma_0)\leq s^R(\sigma_0)\leq s^R(\sigma_1)\leq s^B(\sigma_1)$. $\square$

Proof of Proposition 3. Suppose $\nu_0\leq\nu^*\leq\nu_1$. Then, expected welfare in the first period is decreasing in $s(\sigma_0)$ and increasing in $s(\sigma_1)$.

Because by Proposition 2, $s^B(\sigma_0) \le s^R(\sigma_0)$ and $s^B(\sigma_1) \ge s^R(\sigma_1)$, hindsight bias increases first-period welfare. From the definition of ν^*,

$$\nu_0 \le \nu^* \le \nu_1 \Longleftrightarrow \frac{\nu_0(1+p)-p}{1-p} = \underline{\kappa} \le \kappa \le \frac{\nu_1(1+p)-p}{1-p} = \overline{\kappa}.$$

Now suppose $\nu_0 < \nu_1 < \nu^* \Longleftrightarrow \kappa > \overline{\kappa}$. Then, first-period welfare is decreasing in both $s(\sigma_0)$ and $s(\sigma_1)$. If $p > 0$, hindsight bias has a positive effect in the σ_0 subgame but a negative effect in the σ_1 subgame; the overall effect is ambiguous. If $p = 0$, however, the term in $s(\sigma_0)$ in (9) disappears, implying that $s^B(\sigma_1) = s^R(\sigma_1)$. Hence, hindsight bias has no effect in the σ_1 subgame; its overall effect is positive. Finally, suppose $\nu^* < \nu_0 < \nu_1 \Longleftrightarrow \kappa < \underline{\kappa}$. Then, first-period welfare is increasing in both $s(\sigma_0)$ and $s(\sigma_1)$. If $p > 0$, hindsight bias has a negative effect in the σ_0 subgame but a positive effect in the σ_1 subgame; the overall effect is ambiguous. If $p = 0$, $s^B(\sigma_1) = s^R(\sigma_1)$, so hindsight bias has no effect in the σ_1 subgame; its overall effect is negative. $\square$

Appendix B. Sophisticated voters and reliability

Denote the probability that the state of the world is 1 by π and the probability that the signal matches the underlying state of the world by $x_\omega \equiv Pr(\sigma_\omega | \omega)$. Since $\nu_0 = \frac{\pi(1-x_1)}{\pi(1-x_1)+(1-\pi)x_0}$ and $\nu_1 = \frac{\pi x_1}{\pi x_1 + (1-\pi)(1-x_0)}$, to be consistent with Assumption 1, x_0, x_1 and π must satisfy $(x_0, x_1, \pi) \in (0,1)^3$ and $\frac{1-x_0}{x_1} < \frac{\pi}{1-\pi} < \frac{x_0}{1-x_1}$. We can apply Bayes' rule to obtain

$$r_0 = \frac{x_0(\lambda + (1-\lambda)s(\sigma_0))}{x_0(\lambda + (1-\lambda)s(\sigma_0)) + (1-x_0)(\lambda + (1-\lambda)s(\sigma_1))},$$

$$r_1 = \frac{x_1(\lambda + (1-\lambda)s(\sigma_1))}{x_1(\lambda + (1-\lambda)s(\sigma_1)) + (1-x_1)(\lambda + (1-\lambda)s(\sigma_0))}.$$

Clearly, $0 < r_\omega < 1$, $\omega = 0, 1$, implying that the politician's reputation is a convex combination of those under rational and naive hindsight-biased evaluation and is strictly different from his reputation with either rational or naive hindsight-biased voters.

References

Bénabou, R., Tirole, J., 2002. Self-confidence and personal motivation. Quarterly Journal of Economics 117 (3), 871–915.

Besley, T., 2006. Principled agents? The political economy of good government. Lindahl Lectures on Monetary and Fiscal Policy. Oxford University Press, Oxford.

Blank, H., Musch, J., Pohl, R.F., 2007. Hindsight bias: on being wise after the event. Social Cognition 25 (1), 1–9.

Brulé, D.J., 2005. Explaining and forecasting leaders' decisions: a poliheuristic analysis of the Iran hostage rescue decision. International Studies Perspective 6, 99–113.

Bryant, F.B., Brockway, J.H., 1997. Hindsight bias in reaction to the verdict in the O. J. Simpson criminal trial. Basic and Applied Social Psychology 19 (2), 225–241.

Camerer, C., Loewenstein, G., Weber, M., 1989. The curse of knowledge in economic settings: an experimental analysis. Journal of Political Economy 97 (5), 1232–1254.

Canes-Wrone, B., Herron, M.C., Shotts, K.W., 2001. Leadership and pandering: a theory of executive policymaking. American Journal of Political Science 45 (3), 532–550.

Christensen-Szalanski, J.J.J., Willham, C.F., 1991. The hindsight bias: a meta-analysis. Organizational Behavior and Human Decision Processes 48 (1), 147–168 February.

Compte, O., Postlewaite, A., 2004. Confidence-enhanced performance. The American Economic Review 94 (5), 1536–1557.

Crémer, J., 1995. Arm's length relationships. Quarterly Journal of Economics 110 (2), 275–295.

Downs, G.W., Rocke, D.M., 1994. Conflict, agency, and gambling for resurrection: the principal-agent problem goes to war. American Journal of Political Science 38 (2), 362–380.

Fischhoff, B., 1975. Hindsight ≠ foresight: the effect of outcome knowledge on judgment under uncertainty. Journal of Experimental Psychology. Human Perception and Performance 1, 288–299.

Frey, B., Eichenberger, R., 1991. Anomalies in political economy. Public Choice 68, 71–89.

Guilbault, R.L., Bryant, F.B., Brockway, J.H., Posavac, E.J., 2004. A meta-analysis of research on hindsight bias. Basic and Applied Social Psychology 26 (2&3), 103–117.

Harrington, J.E., 1993. Economic policy, economic performance, and elections. The American Economic Review 83 (1), 27–42.

Hawkins, S.A., Hastie, R., 1990. Hindsight: biased judgments of past events after the outcomes are known. Psychological Bulletin 107 (3), 311–327.

Hess, G.D., Orphanides, A., 1995. War politics: an economic, rational-voter framework. The American Economic Review 85 (4), 828–846.

Hoffrage, U., Hertwig, R., Gigerenzer, G., 2000. Hindsight bias: a by-product of knowledge updating? Journal of Experimental Psychology. Learning, Memory, and Cognition 26 (3), 566–581.

Houghton, D., 2001. U.S. Foreign Policy and the Iran Hostage Crisis. Cambridge University Press, Cambridge.

Köszegi, B., 2006. Ego utility, overconfidence and task choice. Journal of the European Economic Association 4 (4), 673–707.

Levy, G., 2004. Anti-herding and strategic consultation. European Economic Review 48 (503–525).

Majumdar, S., Mukand, S.W., 2004. Policy gambles. The American Economic Review 94 (4), 1207–1222.

Maskin, E., Tirole, J., 2004. The politician and the judge: accountability in government. The American Economic Review 94 (4), 1034–1054.

Mueller, J.E., 1994. Policy and Opinion in the Gulf War. University of Chicago Press, Chicago.

Mullainathan, S., 2002. A memory-based model of bounded rationality. Quarterly Journal of Economics 117 (3), 735–774.

Prat, A., 2005. The wrong kind of transparency. The American Economic Review 95 (3), 862–877.

Rabin, M., 1998. Psychology and economics. Journal of Economic Literature 36 (1), 11–46.

Rogoff, K., 1990. Equilibrium political budget cycles. The American Economic Review 80, 21–36.

Smart, M., Sturm, D.M., 2006. Term limits and electoral accountability, Centre for Economic Performance Discussion Paper no. 770.